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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

EDITED BY

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DECEMBER 1916

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MUTUAL REPULSION OF SPECTRAL LINES AND OTHER
SOLAR EFFECTS CONCERNED WITH ANOMA-
LOUS DISPERSION¹

BY SIR JOSEPH LARMOR

In *Philosophical Transactions of the Royal Society of London*, 189, 240, 1897, which I believe was the earliest attempt at detailed discussion of such matters, the refractive index of a gas in the neighborhood of an *ideally simple* line is given by

$$\frac{\mu^2 - 1}{\mu^2 + 2} = \frac{ng_i}{p_i^2 - p^2}$$

where

$$p_i = \frac{2\pi c}{\lambda_i}, \quad p = \frac{2\pi c}{\lambda};$$

n is the number of molecules per unit volume, g_i a molecular constant for the line (p_i).

The absorption line is black *absolutely* from

$$p^2 = p_i^2 - ng_i \text{ to } p^2 = p_i^2 + 2ng_i;$$

thus its breath δp ($= -\frac{2\pi c}{\lambda_i^2} \delta \lambda$) is given by

$$\frac{\delta \lambda}{\lambda} \left(= -\frac{\delta p}{p} \right) = \frac{3}{2} \frac{ng_i}{p_i^2}.$$

¹ From letter to G. E. Hale, of date April 1916, prompted by S. Albrecht's paper; revised August 26.

It is asymmetric with respect to the emission line by one-sixth of its breadth toward the violet end.

How far from this absorption line is the extra refractive power of the medium, due to the material that produces the line, recognizable? The pressure-effect in a gas is substantially, or largely, an effect of density operating through the increased value of the dielectric coefficient K or μ^2 . The train of ideas in *Astrophysical Journal*, 26, 120, 1907, which of necessity is most rough, only indicating the order of magnitude to be expected, shows that change of μ from 1 to 1.003 in the surrounding gas may be expected to alter the wave-length of a vibration of a molecule immersed in that gas by an amount of the order given by

$$\frac{\delta\lambda}{\lambda} \left(= -\frac{\delta p}{p} \right) = 10^{-6}.$$

This uses the data of Humphreys' original observations. A change in λ of 0.001 A means $\frac{\delta p}{p} = \frac{1}{3} \cdot 10^{-6}$, say, and to produce it would thus require the change in μ to be of the order $\frac{1}{2} \cdot 10^{-3}$. Using this value $\frac{1}{2} \cdot 10^{-3}$ for $\delta\mu$, put then for the gaseous atmosphere

$$\mu^2 = 1 + \varepsilon + 10^{-3},$$

where $1 + \varepsilon$ is ordinary, owing to bands at a distance, and 10^{-3} is local, selective, or anomalous (the value of the constant of astronomical refraction makes ε for air at the bottom of the atmosphere $3 \cdot 10^{-4}$) and the first formula gives, as $\mu^2 + 2$ is practically 3,

$$\frac{10^{-3}}{3} = \frac{ng_i}{p_i^2 - p^2};$$

while the breadth of the absolutely black part of the line is

$$(\delta\lambda)_0 = \frac{3\lambda_i}{2p_i^2} ng_i.$$

This gives

$$p_i^2 - p^2 = 3 \cdot 10^3 ng_i = 2 \cdot 10^3 \frac{p_i^2}{\lambda_i} (\delta\lambda)_0$$

or

$$1 - \frac{\lambda_i^2}{\lambda^2} = 2 \cdot 10^3 \frac{(\delta\lambda)_0}{\lambda_i},$$

giving

$$\frac{\lambda \delta \lambda}{\lambda^2} = 10^3 \frac{(\delta \lambda)_0}{\lambda_1},$$

or say

$$\delta \lambda = 10^{-3} (\delta \lambda)_0.$$

Thus μ is altered in the gas by the assumed amount $\frac{1}{2} \cdot 10^{-3}$ for radiation at a distance $\delta \lambda$ from the line λ_1 causing the alteration, which is given by $\delta \lambda = 10^3 (\delta \lambda)_0$; viz., if $(\delta \lambda)_0$ is the breadth of the part of an absorption line of *simplest type that is absolutely black, the wave-length of an adjacent independent line will be affected to the order 0.001 A if that line is at a distance from it of 1000 $(\delta \lambda)_0$.* The molecules of the gas are supposed to be stationary in this deduction: thus really $(\delta \lambda)_0$ is the very small residue that remains when the Doppler-Fizeau effect of molecular translatory motion is subtracted.

This is very rough and incomplete, and we have had to introduce the unknown $(\delta \lambda)_0$. But Becquerel made observations on the sodium D lines long ago, from which the anomalous change $\delta \mu$ in their neighborhood can be measured directly without doubt: in *Comptes Rendus*, 127, 899, 1898; 128, 146, 1899, quoted by Kelvin in *Philosophical Magazine* (5), 46, 494, 1898, or *Baltimore Lectures*, p. 176, where, however, the investigation is imperfect (except for a gas) and should be replaced by the above. R. W. Wood has, I think, measured the deviation more precisely since. It is of course easy to measure it with adequate apparatus. Using such direct data, the only remaining question is whether the rough hypothesis quoted above from the *Astrophysical Journal* gives the right order of magnitude; it errs, I think, as an underestimate.

I now see that King's flexured spectra give pertinent information. The flexure is due to the light passing across a horizontal trough of vapor, something like a prism, which involves deviations of the order $\mu - 1$ as the angle of the prism is about half a right angle or more. If this angular deviation is about $\frac{1}{2} \cdot 10^{-3}$ (which would give $\delta \lambda$ about 0.001 of the black breadth of the absorption line), it must give rise to a transverse displacement in the spectrum of $\frac{1}{8}$ inch, as it appears on the photograph, if the plate was at a distance of $1000/4$ inches, say 20 feet, from the vapor prism, lenses

in the path being allowed for. Even in that experiment it is thus likely that the density of the vapor was below the limit that could be expected to show a dynamical influence on the wave-length of an adjacent line (separated by 1000 ($\delta\lambda$)₀ from it).

I have now looked up R. W. Wood's paper (*Philosophical Magazine*, 8, 295, 1906) and that of Julius (*Astrophysical Journal*, 25, 95, 1907). The latter gives on p. 99 a table of densities of saturated sodium vapor; at temperature 420°C. the density is only $\left(\frac{0.0013}{0.00007}\right)^{-1}$ or 1/200 of that of atmospheric air. Such figures seem to give no chance of affecting sensibly the wave-length of an adjacent line.

Julius speaks of $\mu-1$ being so very great as 0.36 at 0.4 of an angstrom from a D line, quoting Wood. This would imply deviation of that radiation (0.4 from D) of the order $\mu-1$ radians in getting refracted obliquely out of such a region of sodium vapor. He says nothing about the breadth of Wood's D line, probably far removed from the ideal simple type with which we have been dealing; if it came anywhere near to 0.4 Å, such adjacent light just outside the margin of the line cannot travel far without absorption or scattering in the sodium vapor. In any case its elimination would only produce a blurring of the D line, what, I think, Julius calls a "dispersion band." This does not exist sensibly in the solar spectrum, though it does in Wood's vapor—thus indicating that the density of vapor in the sun is much less than Wood's, which makes it to me unexpectedly, but of course not surprisingly, small and obliterates anomalous influences—extending even to the case of the arc when its lines are sharp, for which the Pasadena laboratory has now a negative result, even in an electric furnace. If one had thought of all the above, the experiment on mutual repulsion of furnace lines might have been unnecessary.

As to the suggested distortion of the form of solar prominences by anomalous refraction: Consider a volume of glowing vapor as represented in Fig. 1. Light emitted from a point A in it is refracted to the observer as if it came from A'. This crude representation exaggerates the effect. Even if the anomalous part of $\mu-1$ were 0.36, according to Wood's measurement, which is

absurd in the present circumstances, the angle ACA' would be only about 20° , so that the abnormal displacement (of A to A') would be less than one-third of the depth AC of the highly refracting vapors, and that in the most favorable case; but under such conditions the path of the light from A to C could not be more than a few miles before extinction¹ arrives, so that the distortion of position (A to A') could not possibly be so much as one-third of this. On the other hand, if the vapor-density is much smaller, so is the angle ACA' , to an extent compensating the longer possible path AC .

The stratification of the vapor must be on the whole parallel to the sun's surface. If then distortions existed, they would be more prevalent and conspicuous and the prominence lines would appear more ragged nearer the limb, where the light issues obliquely, than around the center of the disk.

The abnormal flare of the Sherman F line of 1872, figured in Abbot's *The Sun*, p. 163, is surely broadening owing to the condensation or other affection of the gas which is revealed by the concurrent continuous spectrum.

If anomalously refracted light from A thus appears to come from A' , x miles distant, while the other parts of the light appear to come from the true source A , the spectroscope will receive the former at an inclination x/D to the true line of sight, where D is the sun's distance. If AA' were as much as 10 miles, the inclination would be $1/42$ of a second of arc. This would show itself as an apparent change in the angular dispersion for that constituent of the light, and therefore as an apparent proportionate change of λ , of the same order when a high-dispersion grating is used; it would then amount to about $\frac{1}{2} \cdot 10^{-3}$ of an angstrom in the visible spectrum.² On the other hand, the first estimate made above refers to a true change of wave-length (or period) of the source, arising from the influence of neighboring sources of adjacent wave-length. The considerations advanced by Professor W. H. Julius (*Astrophysical*

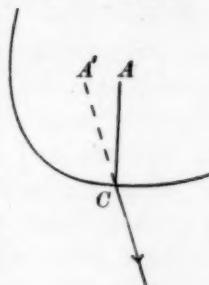


FIG. 1

¹ See footnote *infra*.

Journal, 43, 43, 1916) refer to deviation of rays in the solar atmosphere by anomalous refraction without change of period; these apparently are subject to the extreme estimate just now made, and thus seem to be beyond present instrumental means.¹

The considerations *supra* as to change of period hold good also for influence of an adjacent line of the *same* substance, provided this line arises from a separate independent vibration. A very searching test would be afforded by the behavior of the satellites of a line, when the vapor-density of the source is increased; but a negative result can be taken as an indication that the satellite is an essential part of the vibration of the main line, and not due to a different source which can be varied independently of it; cf. Evershed's observations, recently reported in *The Observatory*, 39, 59, 1916, and the suggestion on these lines that followed in connection therewith.

The case of rays part of whose path is tangential to the gaseous strata of density needs further consideration. By James Thomson's principle, the curvature of a ray in a heterogeneous medium is $d \log \mu/dn$, where dn is measured along its normal drawn inward. For a gas this is practically $d\mu/dn$; and if the ray travels for a considerable distance nearly tangential to a stratum across which the gradient of density is very steep, its total deviation will be large, giving rise to possibly hundreds of times the $\frac{1}{2} \cdot 10^{-3}$ of an angstrom mentioned above. But would it be shown by the spectroscope?

Suppose the curves of the diagram (Fig. 2) to represent the stratification of density in the mass of gas. The light from the different point-sources, in or behind the mass, which is caught up by the spectroscope, is a narrow ray from each point, all parallel. Where the ray is tangential to a surface of stratification it is drawn around toward it, as regards that constituent of frequency susceptible to anomalous refraction by the gas, while the rest passes

¹ If I am not under a misapprehension in this intricate subject, it seems to follow that true change of period, such as arises from line-of-sight motion, implies angular displacement of the lines increasing in proportion to the dispersive power of the spectroscope that is employed, while mere anomalous deviation of the rays, without change of period, would give the same angular deviation for all dispersive powers.

on; but elsewhere the paths are practically straight. If in its path the ray is anywhere tangential to a stratum,¹ its anomalous constituent is thus thrown off the slit of the spectroscope; but its place is taken there by the same constituent from a source more to the right in the diagram, by a few hundreds of miles at the very most (cf. *supra*), owing to the shallowness of the solar atmosphere. The light thus caught by the telescope from this source, at one side of the source at which it is pointed, is not quite parallel to the main path of the light; and this difference of direction would come out as a displacement of the spectral line, which might amount to 10^{-2} of an angstrom, thus simulating motion of the original source in the line of sight. But the spectroscope could hardly show it.

The smallest available breadth of the slit represents hundreds of miles in the region of the sun whose image is thrown on it. Thus, for these phenomena to be observable without entire loss of sharpness, the parallel rays from a band of sources hundreds of miles wide must be deflected to nearly the same extent where each is tangential to a stratum. This necessary regularity requires that the gradient of density should remain about the same, and of substantial amount along hundreds of miles across the gas; and, in order to account for the observed shifts of lines, bearing in mind the small atmospheric pressure notwithstanding high gravity (cf. the remark on scale, *infra*), this would seem to pile up the density of the gas of the solar atmosphere to quite impossible values.

Actually in the higher levels the densities are for various reasons probably almost infinitesimally small.

¹ It cannot be anywhere tangential to the strata if its source is in the front half of the mass of gas.



FIG. 2

A general grasp of these matters, in their relation to laboratory experience, is perhaps facilitated by use of a principle which follows from the formula for curvature of the rays, viz., that the deviations of rays in the solar atmosphere are the same as they would be in a model atmosphere, on a scale reduced uniformly in all three dimensions of space, but with the same densities of the gaseous constituents at all corresponding points.

CAMBRIDGE, ENGLAND

August 26, 1916

THE VARIATIONS IN SPECTRAL TYPE OF TWENTY CEPHEID VARIABLES¹

BY HARLOW SHAPLEY

A considerable amount of material relative to the spectral types of variable stars of the Cepheid class has been accumulated at the Mount Wilson Observatory during the past year, chiefly with the aid of the 10-inch photographic telescope. Earlier work on some of these stars at the Lick, Harvard College,² and Pulkova Observatories had shown or suggested variations in certain characteristics of the spectra. The data collected in the present paper, however, indicate that distinct changes in spectral type, accompanying regularly the periodic variations in light and apparent velocity, constitute one of the general and fundamental properties of Cepheid variables. A statement of the Cepheid problem at the time it was taken up at Mount Wilson is outlined in *Contribution* No. 92,³ together with a summarized account of the most relevant previous work. The significance of spectral variations in the interpretation of variability in light and velocity is noted in the same article and in subsequent papers on the light-curves and spectra of Cepheid variables.⁴ The following pages will be devoted to the observations of the spectra of individual stars.

A list of all the Cepheids for which definite variations of spectral type have been observed is given in Table I. The positions in the second and third columns are from *Harvard Annals*, 56, No. 6, Table VIII. The data for the light-variations are taken from the same source except that the magnitudes for RT Aurigae are by

¹ *Contributions from the Mount Wilson Solar Observatory*, No. 124.

² See, for instance, the remark by Miss Cannon on the spectrum of δ Cephei in *Harvard Annals*, 56, 110, 1912.

³ *Astrophysical Journal*, 40, 448, 1914.

⁴ *Mt. Wilson Contr.*, Nos. 104, 112; *Astrophysical Journal*, 42, 148, 1915; 43, 217, 1916; *Mt. Wilson Communications*, Nos. 14, 21, 22, 27; *Proceedings of the National Academy of Sciences*, 1, 452, 1915; 2, 132, 136, 208, 1916.

Kiess,¹ those for RR Lyrae by Shapley,² and those for RS Boötis and XZ Cygni from Hartwig's *Ephemeris*.³ Precise information as to the maximum magnitude and range of variation of these stars, even of the much observed brighter ones, is unfortunately not available.⁴ Widely differing values are given in the catalogues by different observers. The uncertainty arises partly from probable

TABLE I
LIST OF SPECTRAL VARIABLES

STAR	R.A. 1900	DECL. 1900	VISUAL LIGHT		PERIOD	SPECTRAL VARIATION	NO. OF PHOTO- GRAPHS
			Maximum	Range			
TU Cassiopeiae..	0 ^h 20 ^m 9	+50° 44'	7 ^m 2	1 ^m 4	2 ^d 139	F1-F8	5
SU Cassiopeiae..	2 43.0	+68 28	5.9	0.4	1.950	A8-F5	24
SZ Tauri.....	4 31.4	+18 20	7.2	0.5	3.149	A9-F7	13
T Monocerotis.	6 19.8	+7 8	5.7	1.1	27.012	F5-G2	9
RT Aurigae.....	6 22.1	+30 34	5.1	0.9	3.728	A7-G1	30
W Geminorum.	6 29.2	+15 24	6.7	0.8	7.916	F2-G1	9
RS Boötis.....	14 29.2	+32 11	9.2	1.0	0.377	B8-F0	13
X Sagittarii..	17 41.3	-27 48	4.4	0.6	7.012	F1-G5	24
Y Ophiuchi.....	17 47.3	-6 7	6.1	0.4	17.113	F5-G3	8
W Sagittarii..	17 58.6	-29 35	4.3	0.8	7.595	A8-G2	25
Y Sagittarii..	18 15.5	-18 54	5.4	0.8	5.773	F4-G4	10
RR Lyrae.....	19 22.3	+42 36	6.8	0.9	0.567	B9-F2	17
U Aquilae.....	19 24.0	-7 15	6.2	0.7	7.024	F6-G2	4
XZ Cygni.....	19 30.4	+56 10	8.7	0.6	0.467	A0-A6	2
U Vulpeculae..	19 32.3	+20 7	6.5	1.1	7.990	F7-G5	8
SU Cygni.....	19 40.8	+29 1	6.2	0.8	3.846	A6-F7	17
η Aquilae.....	19 47.4	+0 45	3.7	0.8	7.176	A8-G5	29
S Sagittae....	19 51.5	+16 22	5.5	0.6	8.382	F4-G3	18
T Vulpeculae..	20 47.2	+27 52	5.5	0.6	4.436	A9-G1	17
δ Cephei.....	22 25.4	+57 54	3.5	0.8	5.366	F0-G2	46

irregularities in the variations, but chiefly because most of the light-observations have been naked-eye comparisons with stars for which various estimates of magnitudes have been adopted. The values in the fourth and fifth columns, therefore, serve in most cases only to indicate approximately the visual brightness and range of light-change. The point is of some importance in that it shows the

¹ *Laws Observatory Bulletin*, No. 23, 1915.

² *Mt. Wilson Contr.*, No. 112; *Astrophysical Journal*, 43, 217, 1916.

³ *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 287, 301, 1914.

⁴ Exceptions are δ Cephei, RR Lyrae, and perhaps a few others. See note to Table VIII below.

futility of attempting at present to study the relation of range in light to range in spectral variation. The data are sufficient, however, to show the relation of spectral type to period (which had already been noted for a larger number of stars on the basis of the Harvard classification),¹ although in a number of cases the extreme range of spectrum is not recorded when observations near maximum or minimum light are wanting.

Of the 328 photographs of spectra used in deriving the results for the stars in Table I, 311 were made by the writer with the 10-inch portrait lens and objective prism. The remainder were made with the 60-inch reflector—13 by Mr. Pease and 4 by Mr. Adams. For nearly all the work with the 10-inch, a 15° prism is employed. The refracting edge is set perpendicular to the hour circle, and the spectrum, drifting in right ascension, is widened as desired by adjusting the driving clock. For some of the brighter variables, a small 30° prism was used on a few nights in conjunction with the 15° prism. Such instances are designated by the note " 45° " in the columns of remarks in Tables III-XIX. Except in the cases noted, all photographs are on Seed "27" plates.

With the single prism the dispersion is $H\beta - H\epsilon = 5.2$ mm; for the two prisms $H\beta - H\epsilon = 12$ mm. The uncertainty of a determination of spectral type with the lower dispersion is one- or two-tenths of a spectral interval, but there may be a small systematic difference between my classification and that made at Harvard. So far as possible, however, the choice of criteria for type has been based upon the classifications of the *Draper Catalogue*.

In preparing the material for tabular presentation, all the spectra were classified before phases were computed. This procedure lessens the possibility of errors of prejudice where small changes are concerned. Only two or three discordant results remained after some errors of identification were eliminated, and such discordances may be due to irregularities in the variations or to erroneous light-elements.

As remarked in earlier papers, the changes observed in the spectra of these stars are between known, normal types. No peculiarities appear other than those to be expected in the spectra

¹ *Mt. Wilson Contr.*, No. 92, p. 16; *Astrophysical Journal*, 40, 463, 1914.

of stars of very high luminosity. Possible exceptions are the spectrum of W Geminorum, also noted at Harvard,¹ and the occasional unequal sharpness of the hydrogen lines in spectra made during increasing light. With high dispersion other peculiarities may appear, as, for instance, those indicated in the special study of δ Cephei.²

Continuous and periodic changes of spectral type are shown distinctly for fifteen of the stars over several epochs of maximum, and are definitely indicated for the remainder of the list. No Cepheid variable observed has failed to show variability of spectrum. Upon the basis of this result we may believe that for the two or three thousand variable stars which belong to the same class similar disturbances of the radiating surfaces underlie similar periodic oscillations in light and spectral type.

The variation in spectrum of a Cepheid is undoubtedly as important a part of the phenomenon as the fluctuation in light; moreover, it should be as definite a method of detecting a star's peculiar variability as the measures of magnitude. By collecting the classifications of spectrum into normals (using the period that defines the variation in light) and plotting the mean spectrum against phase, we may obtain a curve closely similar to the curves of variation in magnitude and velocity. Such a curve is shown for δ Cephei in Fig. 1, based upon the data in Table II. It differs in form from the light-curve on a later page in certain respects—the spectrum falls away from its maximum (bluest) value more rapidly than the light falls from its maximum, and the change from red to blue spectrum at the end of minimum light is frequently abrupt. To some extent this latter result probably reflects the oscillation in the time of the rise to maximum light, which has been shown to be conspicuous for several Cepheids.

The changes in the spectrum of RR Lyrae are discussed in *Contribution No. 112*. Data for RS Boötis are given by Pease in *Publications of the Astronomical Society of the Pacific*, 26, 256,

¹ *Harvard Annals*, 55, 38, 1907.

² *Mt. Wilson Communications*, No. 22; *Proceedings of the National Academy of Sciences*, 2, 136, 1916.

1914.¹ The observations of XZ Cygni by Adams will be published elsewhere. For the other seventeen stars listed in Table I the variations are detailed in the accompanying series of tables and figures.

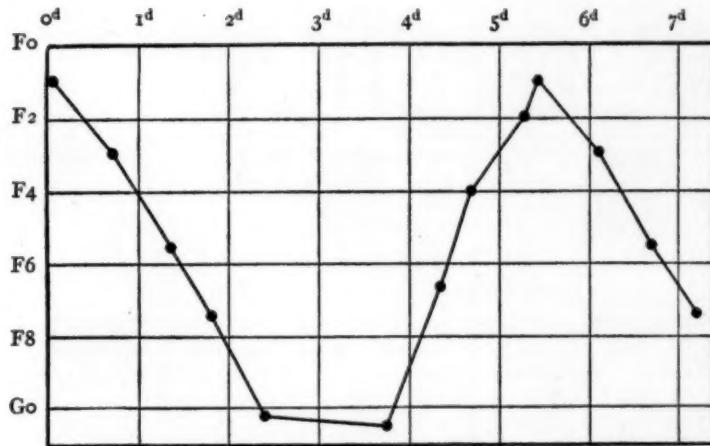


FIG. 1.—The curve of spectral variation of δ Cephei

TABLE II
SPECTRAL VARIATION OF δ CEPHEI

Mean Phase	Mean Spectrum	Number Spectra	Weight
0.04	F1.0	7	12
0.74	F3.0	6	11
1.36	F5.6	5	8
1.82	F7.5	4	8
2.40	Go.2	4	8
3.76	Go.5	4	8
4.37	F6.7	4	7
4.72	F4.0	5	10
5.29	F1.9	7	14

¹ A later curve of the spectral variation is shown in *Contribution* No. 112 and *Communication* No. 21.

TABLE III
TU CASSIOPEIAE
Max. = J.D. 2419302.12 + 2^d139·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
71.....	1916 Feb. 2	9 ^h 21 ^m	13 ^m	1 ^d 048	F6	Faint
85.....	5	9 46	18	1.926	Fo:	
100.....	6	8 44	25	0.744	F1	
		9 04	16	0.758	F2	
108.....	7	7 40	37	1.704	F8	

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 334, 1914. The visual range given by Hartwig is 1^M1.

TABLE IV
SU CASSIOPEIAE
Max. = J.D. 2417287.30 + 1^d9498·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
48.....	1915 Dec. 9	12 ^h 04 ^m	8 ^m	0 ^d 051	F1	
61.....	1916 Jan. 31	8 23	10	0.253	Fo:	Out of focus
		36	10	0.262	Fo:	
		46	10	0.269	Fo:	
62.....	31	10 36	11	0.345	F3	
		46	11	0.352	F4	
		56	6	0.359	F2:	
70.....	Feb. 2	8 55	6	0.325	F5:	
		9 05	12	0.332	F5	
81.....	5	6 56	15	1.292	A9	
		7 09	10	1.301	A8	
87b.....	5	11 42	11	1.491	F2	
		50	5	1.497	Fo:	
		59	10	1.503	Fo	
101.....	6	9 34	12	0.453	F2	
		46	8	0.461	F1	
		53	4	0.466	F1	
109.....	7	8 27	12	1.406	A9	
		39	10	1.415	Fo	
		52	15	1.424	F2	
110.....	7	9 09	14	1.435	F1	Clouds
		24	14	1.446	Fo	
		42	21	1.458	Fo	

* The phases are computed with the elements derived by Müller and Kempf (*Astronomische Nachrichten*, 173, 307, 1907). The agreement with spectral type is not good, but the difficulty probably is attributable to inaccurate light-elements rather than to an anomalous relation between spectrum and light-variations. Using the later elements by Parkhurst (*Astrophysical Journal*, 28, 279, 1908):

Max. = J.D. 2417287.30 + 1^d9490·E,
the agreement is equally bad; but with an intermediate period, 1^d94935, the representation is all that could be expected when the small range and the quality of the observations are considered. Direct observations of the brightness of the spectral images on the plates in Table IV verify the supposition that the light-elements are erroneous.

TABLE V

SZ TAURI

Max. = J.D. 2410000.60 + 3^d1487. E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
26.....	1915 Nov. 14	9 ^h 46 ^m	180 ^m	0 ^d 356	F ₂	Overexposed
	Dec. 12	14 59	60	0.235	F ₀	
		15 39	7	0.262	A ₉	
63.....	1916 Jan. 31	11 08	5	2.844	F ₃ :	High wind
		18	10	2.851	F ₄ :	
	31	13 11	8	2.929	F ₀ :	
64.....		22	10	2.937	F ₂	Field low
	Feb. 5	12 20	17	1.597	G	
	6	6 36	3	2.358	F ₆	
88.....		42	7	2.362	F ₇	Faint
	97.....	10 02	3.5	2.501	F ₅	
		21	30	2.514	F ₆	
102.....	6	12 16	33	0.446	F ₀	
113.....						

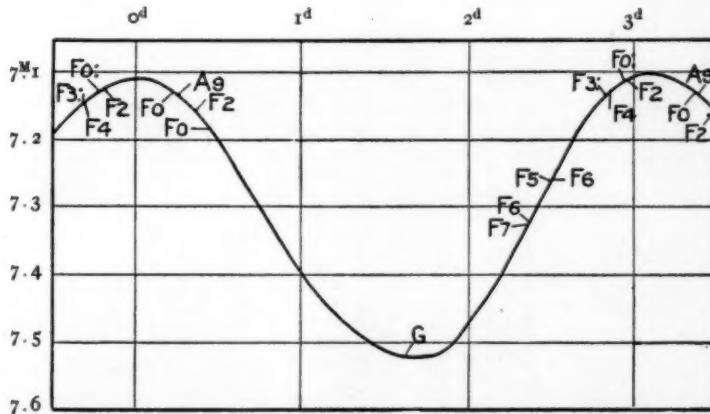
* Leavitt, *Harvard Circulars*, No. 186, p. 2, 1914.

FIG. 2.—SZ Tauri. Mean light-curve by Miss Leavitt (*op. cit.*). An increase of five seconds in the length of the period (a change well within the uncertainty of the light-elements) would place the plotted spectral observations a fourth of a day earlier and harmonize better with the light-curve.

TABLE VI
T MONOCEROTIS
Max. = J.D. 2409633.63 + 27⁴0122·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
78.....	1916 Feb. 2	13 ^h 34 ^m 45	10 ^m 10	26 ^d 193 26.201	F8 F8	
89.....	5	13 06	20	2.162	F6	Wind
111.....	7	11 02	25	4.076	F6	Clouds
114.....	7	12 54	33	4.153	F7	Clear
121.....	Apr. 7	8 41	30	9.953	G2	
160.....	25	9 26	5	0.972	F5	Wind
169.....	27	8 54	16	2.950	F6	
174.....	28	8 58	29	3.953	F8	Clouds

* Yendell, *Astronomical Journal*, 16, 149, 1896.

TABLE VII
RT AURIGAE
Max. = J.D. 2417173.459 + 3^d72806·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
65.....	1916 Jan. 31	13 ^h 38 ^m 46	4 ^m 10	0 ^d 839 0.844	F2 F2	
67.....	Feb. 1	6 31	10	1.542	F7	Plate fogged
77.....	2	12 56	10	1.549	F5	
		13 03	4	2.810	F7	
		11	10	2.815	Go	
90.....	5	13 47	11	2.117	Go	
		57	10	2.124	G1	
104.....	6	11 38	12	3.027	Go	
112.....	7	11 28	13	0.292	A8	
		38	5	0.299	A8	
119.....	Mar. 30	11 14	9	0.090	A7	
		22	8	0.095	A8	
120a.....	Apr. 7	7 35	7	0.481	Fo	
120b.....	7	7 48:	4:	0.490:	A9	Time uncertain
		55	8,	0.495	A9	
122.....	7	9 10	17	0.547	F1	
145.....	9	10 04	7	2.585	F8	
		11	6	2.590	F8	
146.....	9	10 21	10	2.597	F7	Faint
159.....	25	9 01	8	3.029	Fo	
		08	5	3.634	F1	
		11	2	3.636	A9	
167.....	26	7 30	3	0.838	F4	Strong twilight
168.....	27	7 24	10	1.834	F8:	Faint
170.....	27	9 15	12	1.911	F9	
		24	5	1.917	F8	
175.....	28	9 21	12	2.916	F7	
		38	21	2.927	F6	

* Kless, *Law Observatory Bulletin*, No. 23, 1915.

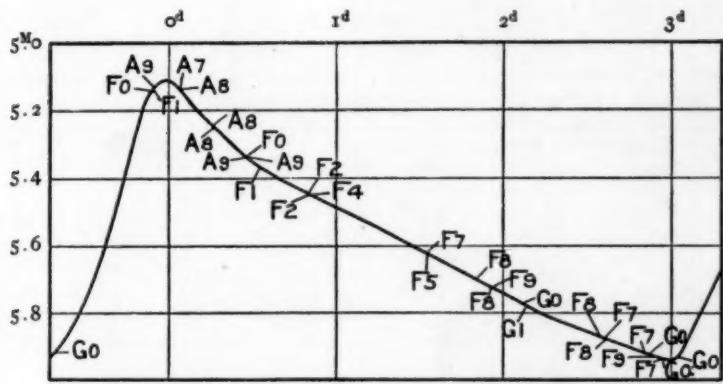


FIG. 3.—RT Aurigae. Mean visual light-curve by Kiess (*op. cit.*), neglecting supposed oscillations in the descending branch.

TABLE VIII

W GEMINORUM

Max. = J.D. 2413266.34 + 7⁴91603 · E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
78.....	Feb. 2	13 ^h 34 ^m	10 ^m	7 ^d 422	F ₂	Haze
		45	10	7.429	F ₂	
89.....	5	13 06	20	2.486	F7:	Wind; faint
103.....	6	11 16	20	3.410	G	Faint
111.....	7	11 02	25	4.400	F6:	Clouds
114.....	7	12 54	33	4.478	G ₀	Clear
121.....	Apr. 7	8 41	30	0.974	F ₄	
169.....	27	8 54	16	5.151	G ₁	
174.....	28	8 58	29	6.153	F ₃	Haze

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 334, 1914. The visual range in Table I is $0^m 8$; according to Hartwig it is $1^m 3$, and from unpublished observations made at Utrecht and generally placed at my disposal by Dr. Van der Bilt it appears to be less than $0^m 4$.

TABLE IX
X SAGITTARII
Max. = J.D. 2403169.93 + $7^{\text{d}} 01 \text{m} 19\text{s}$ E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
69.....	1916 Feb. 1	17 ^h 34 ^m 38	3 ^m 5	0.052 0.055	F2 F3	Dawn
96.....	Apr. 5	17 36	7	4.053	G	
130.....	Apr. 7	15 49 57 16 03	10 5 6	2.872 2.878 2.882	F8 F8 F6:	
140.....	8	13 22	7	3.770	F8:	
156.....	9	15 39	14	4.865	F8	
158.....	9	16 24	10	4.897	F8	
164.....	25	15 43 50	10 5	6.844 6.849	F1	
183.....	28	14 14 20	4 9	2.770 2.774	F7 F6	
187.....	July 9	11 36 48	13 8	4.542 4.550	G1 G1	
198.....	28	9 51	10	2.433	F4	
205.....	29	10 00:	10	3.439:	F8:	45°, faint
219a.....	30	10 10	23	4.446	G5	45°, faint
220.....	30	10 52	20	4.475	G4	45°, faint
236.....	31	11 47	38	5.513	F5	45°
244.....	Aug. 1	8 59	26	6.397	F1	45°, Hy sharp
256.....	2	11 42	34	0.498	F	45°, faint
261.....	3	8 03 16	8 18	1.346 1.355	F3 F2	45°, faint

* Hellerich, *Dissertation*, p. 9 (Bonn, 1913).

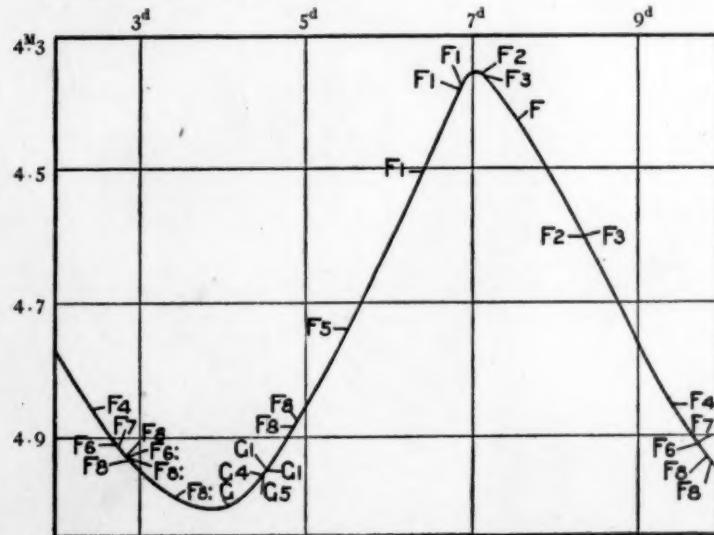


FIG. 4.—X Sagittarii. Mean light-curve by E. C. Pickering (*Harvard Annals*, 46, 155, 1903).

TABLE X
Y OPHIUCHI
Max. = J.D. $2408697.25 + 17^d 11^m 30^s$ E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
68.....	1916 Feb. 1	17 ^h 14 ^m 23	7 ^m 10	14 ^d .35 14.35	G0 F8	Field low Weak
94.....	5	16 22 40	21 15	1.20 1.21	F5 F7	Field low
131.....	Apr. 7	16 16 22	5 10	11.85 11.86	G2 G3	Dawn
139.....	8	13 00	9	12.72	G	
181a.....	28	13 26	10	15.62	F6	Field low

* Hellerich, *op. cit.*, p. 11.

TABLE XI
W SAGITTARII
Max. = J.D. $2403198.88 + 7^d 59^m 43^s$ E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
96.....	1916 Feb. 5	17 ^h 36 ^m	7 ^m	6 ^d 267	F5:	Dawn
130.....	Apr. 7	15 49 57	10 5	7.438 7.444	F0 F0	
140.....	8	16 03 13 22 31 40	6 7 2 5	7.448 0.742 0.748 0.754	A9 F2 F1 F2	
156.....	9	15 39	14	1.837	F4	
158.....	9	16 24	10	1.868	F4	
164.....	25	15 43 50	10 5	2.651 2.656	F5 F6	
183.....	28	14 14 20	4 9	5.589 5.593	G0 G2	
187.....	July 9	11 36 48	13 8	1.535 1.544	F6 F6	
198.....	28	9 51	10	5.274	G1	45°
205.....	29	10 00:	10	6.280	F8	45°, time uncertain
219a.....	30	9 58:	1:	7.278:	A8	45°, time uncertain
220.....	30	10 10	23	7.287	F0	
236.....	31	10 52	20	7.316	A9	45°
244.....	Aug. 1	11 47 8 59	38 26	0.760 1.643	F1	45°
256.....	2	11 42	34	2.756	F3	45°
261.....	3	8 03 16	8 18	3.604 3.613	F6 F9	45°, faint

* Hellerich, *op. cit.*, p. 9.

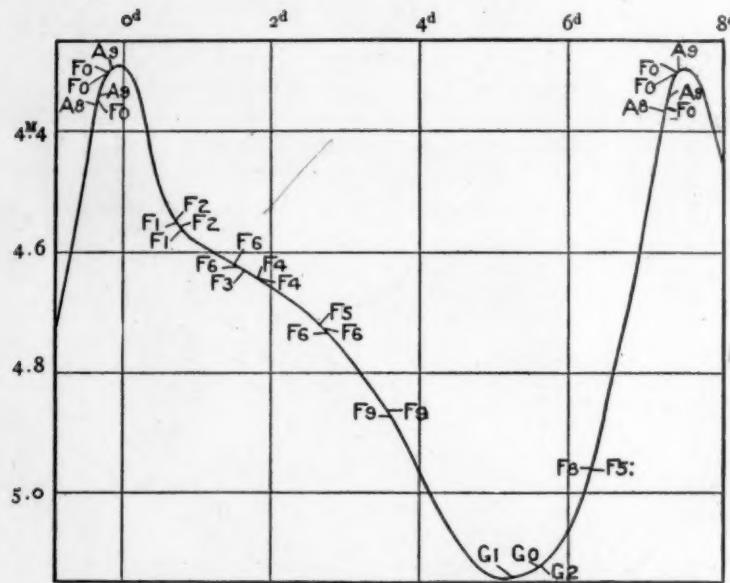


FIG. 5.—W Sagittarii. Mean light-curve by E. C. Pickering (*Harvard Annals*, 46, 155, 1903).

TABLE XII

Y SAGITTARII

Max. = J.D. 2410175.08 + 5^d773268 · E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
130.....	1916 Apr. 7	15 ^h 49 ^m	10 ^m	2 ^d 448	G0	Diffuse
		57	5	2.453	F8:	
		16 03	6	2.458	F8	
140.....	8	13 22	7	3.346	G2	Wind; diffuse
164.....	25	15 43	10	3.124	G:	
183.....	28	14 14	4	0.289	F4	
		20	9	0.293	F4	
187.....	July 9	11 36	13	2.900	F8	Moon near field
261.....	Aug. 3	48	8	2.908	G	
		8 16	18	4.668	G4	

* Hellerich, *op. cit.*, p. 10.

TABLE XIII
U AQUILAE
Max. = J.D. 2410170.325 + 7⁴02387 · E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
142.....	1916 Apr. 8	15 ^h 33 ^m	15 ^m	3.992	G ₂	
181b.....	28	13 48	12	2.847	F ₆	
191.....	July 10	10 06	17	5.455	F ₈	
230.....	31	8 14	17	5.305	F ₆	Diffuse

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 335, 1914.

TABLE XIV
U VULPECULAE
Max. = J.D. 2414200.253 + 7⁴98950 · E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
128.....	1916 Apr. 7	14 ^h 38 ^m	21 ^m	2.573	F ₇	
		54	8	2.584	F ₇ :	Narrow
141.....	8	14 30	27	3.567	G ₄	Diffuse
152.....	9	13 46	20	4.537	G ₄ :	Faint
153.....	9	14 14	15	4.556	G ₆	
161.....	25	13 31	12	4.547	G ₅	
178.....	28	12 25	20	7.501	F _{8p}	H _B strong
211.....	July 29	14 50	22	3.728	G ₅	Difficult

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 335, 1914.

TABLE XV
SU CYGNI
Max. = J.D. 2414202.820 + 3⁴845612 · E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
128.....	1916 Apr. 7	14 ^h 38 ^m	21 ^m	2.383	F ₆	
		54	8	2.394	F ₇	Drift in declination
141.....	8	14 30	27	3.377	A ₆	
152.....	9	13 46	20	0.501	F ₁	
		58	5	0.510	F ₀ :	Narrow
161.....	25	13 31	12	1.108	F ₃	
178.....	28	12 25	20	0.217	A ₈	
184b.....	July 9	9 42	15	2.883	F ₅	
		56	11	2.892	F ₅ :	
188.....	10	8 21	10	3.826	A ₆	
		38	24	3.838	A ₇	
211.....	29	14 33	12	0.011	A ₈	
		50	22	0.023	A ₉	
216.....	30	8 34	17	0.637	F ₄	
231.....	31	8 38	21	1.640	F ₆	
258.....	Aug. 2	13 26	21	0.119	A ₈	
262.....	3	8 40	20	0.920	F ₂	

* Luizet, *Astronomische Nachrichten*, 173, 196, 1906.

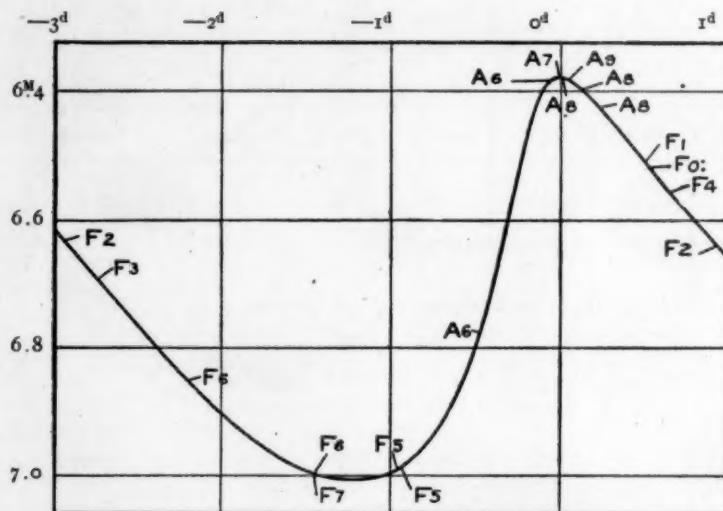


FIG. 6.—SU Cygni. Mean light-curve from observations made at Utrecht; data furnished in manuscript by Dr. Van der Bilt.

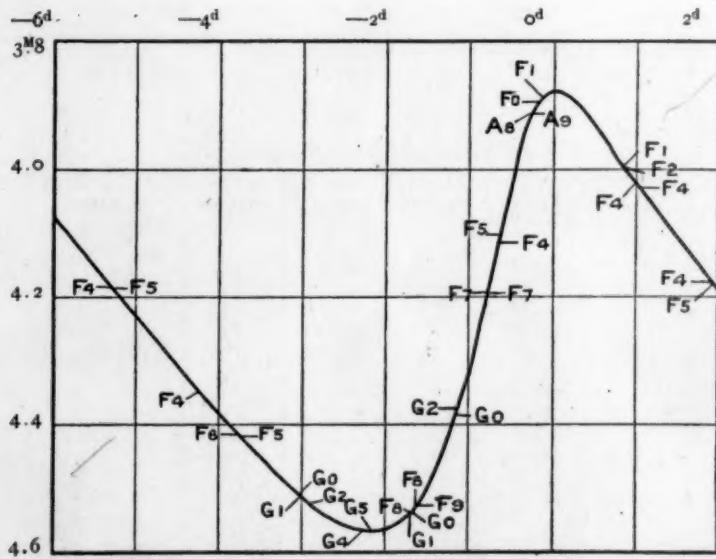


FIG. 7.—η Aquilae. Mean light-curve from observations made at Utrecht; data furnished in manuscript by Dr. Van der Bilt.

TABLE XVI
η AQUILAE
 Max. = J.D. 2396168.732 + 7^d176382·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
142.....	Apr. 8	15 ^h 33 ^m	15 ^m	7.053	F0	
		44	5	7.061	F1	
162.....	25	13 40	3	3.418	F5	
		43	2	3.420	F6	
172b.....	27	14 40	3	5.460	G0	
		42	2	5.461	G1	
181b.....	28	13 40	4	6.418	F7:	Diffuse
		48	12	6.424	F7	
184a.....	July 9	9 16	7	5.471	F8	
		21	4	5.474	F8	
		26	5	5.478	F9	
190.....	10	9 45	15	6.491	F4	
191.....	10	10 06	17	6.506	F5	
196.....	28	8 49	10	-2.923	F4	45°
210.....	29	13 44	9	4.128	G0	45°
		54	11	4.135	G1	
213.....	29	15 43	10	4.210	G2	
215.....	30	8 12	5	4.897	G4	
		17	4	4.901	G5	
230.....	31	8 02	5	5.890	G6	
		14	17	5.899	G2	
243.....	Aug. 1	8 20	2	6.903	A8	45°
		31	12	6.910	A9	
254.....	2	9 47	10	0.787	F1	
		10 01	18	0.797	F2	45°
259b.....	2	14 04	7	0.966	F4	
		12	7	0.971	F4	
269.....	3	13 00	5	1.921	F5	45°
		08	11	1.926	F4	

* Hellerich (*op. cit.*, p. 6) finds that the elements derived by Luizet (*Astronomische Nachrichten*, 163, 361, 1903) are satisfactory without the use of the harmonic term adopted by the latter.

TABLE XVII
S. SAGITTAE

Max. = J.D. 2409863.324 + 8d 381613 · E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
144.....	1916 Apr. 8	16 ^h 18 ^m	11 ^m	2.433	F5	
		26	6	2.438	F6	Dome
153.....	9	14 14	15	3.347	F9	
172a.....	27	14 28	7	4.593	G1	
		34	5	4.597	G2	
179.....	28	12 42	9	5.519	G3	
		50	5	5.525	G2	
184c.....	July 9	10 12	10	1.981	F7	
		20	6	1.987	F6	
		24	3	1.990	F6	
189.....	10	9 07	26	2.936	F8	
		24	6	2.948	F7	
212.....	29	15 11	12	5.426	G2	
227.....	30	15 16	27	6.429	G	
233b.....	31	9 38	10	7.194	F6	45°, faint
		44	1	7.199	F5	
237.....	31	12 41	51	7.322	F6	Wind
259a.....	Aug. 2	13 50	15	0.488	F4	45°
						Very thick

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 335, 1914.

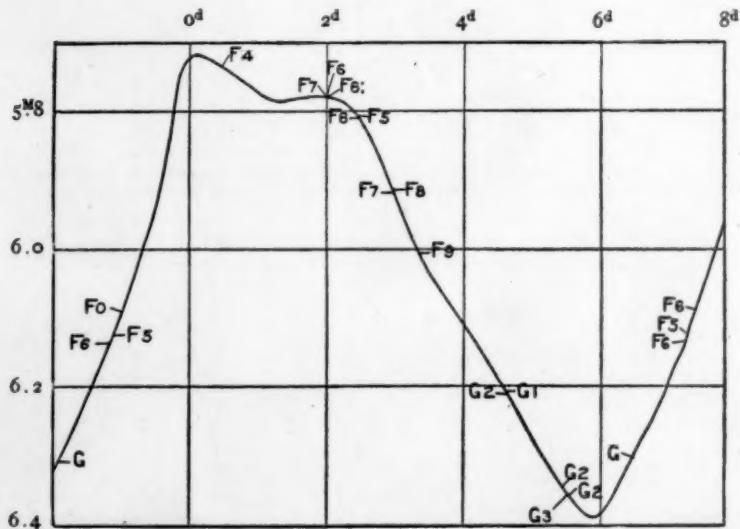


FIG. 8.—S Sagittae. Mean light-curve by Gore as reduced by Luizet (*Astronomische Nachrichten*, 168, 349, 1905).

TABLE XVIII
T VULPECULAE

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
154.....	Apr. 9	14 ^h 31 ^m	10 ^m	3.918	F4	
		40	7	3.924	F5	
163.....	25	13 54	7	2.150	G1	
		14 00	5	2.154	G0	
171.....	27	14 12	8	4.162	F0	
		18	3	4.167	A9	
180.....	28	13 02	8	0.679	F4	
		10	9	0.685	F3	
186a.....	July 9	10 56	9	1.623	F6	
192.....	10	10 26	3	2.602	F7	
		33	8	2.607	F8	
214.....	29	15 56	12	4.089	F2	Hδ very faint
217.....	30	8 52	3	0.359	F0	Hδ stronger
		9 00	11	0.365	F0	
233a.....	31	9 22	15	1.380	F5	
260.....	Aug. 2	14 20	3:	3.587:	F6	Time uncertain
		40	9	3.601	F6	Thick

* Hellerich, *op. cit.*, p. 7.

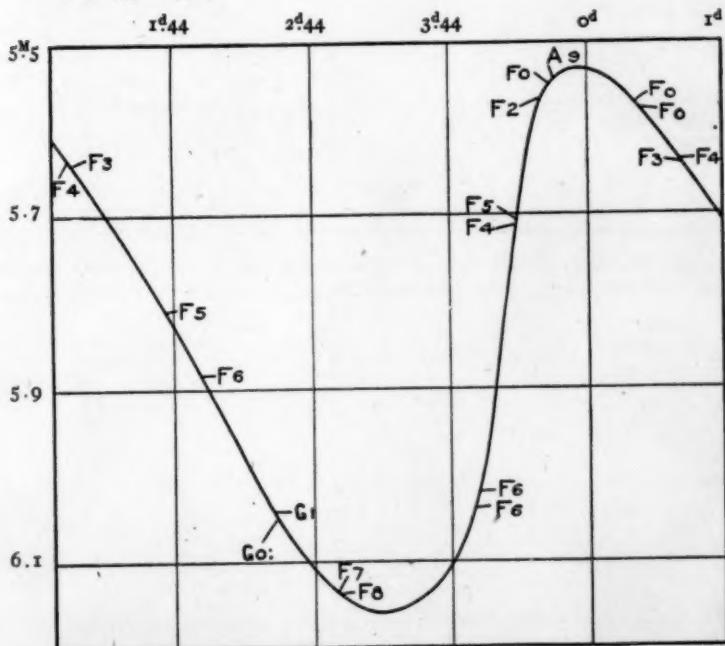


FIG. 9.—*T Vulpeculae*. Mean light-curve by E. C. Pickering (*Harvard Annals*, 46, 156, 1903).

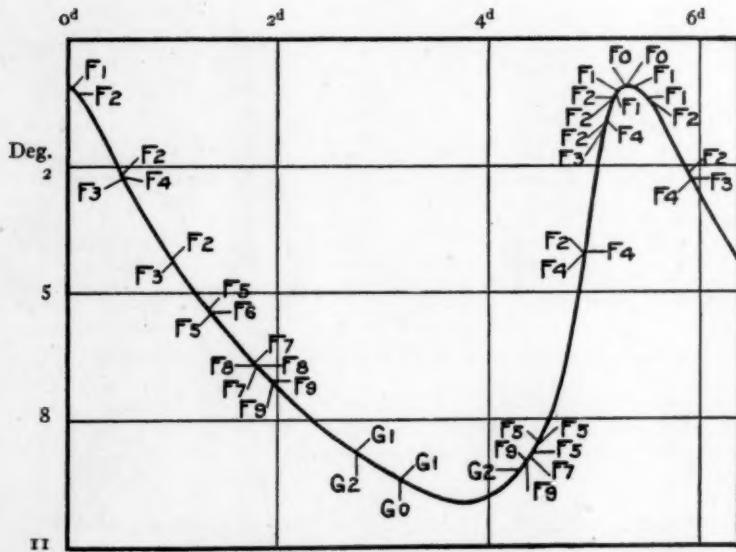


FIG. 10.— δ Cephei. Mean light-curve by Luizet (*op. cit.*). The curve printed in *Popular Astronomy*, 25, 355, 1916, has been revised and supplemented by later data.

TABLE XIX
 δ CEPHEI
 Max. = J.D. 2393659.856 + 5^d366386·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
Y 4571...	Dec. 23	7 ^h 04 ^m	225 ^m	4 ^d 294	G ₂	60-inch reflector
Y 4578...	24	14 07	120	5.254	F ₄	60-inch reflector
	1916					
60.....	Jan. 31	7 28	5	0.379	F	Bad focus
66.....	Feb. 1	6 59	4	1.359	F ₅	Haze
		7 02	1	1.361	F ₅	
		04	3	1.362	F ₆ :	
		14	4	1.369	F ₆	
80.....	5	6 28	3	5.338	F :	Clouds
		32	3	5.341	F ₁	
		34	3	5.342	F ₀	
83.....	5	8 18	2	0.048	F ₁	
		20	2	0.049	F ₂	
98.....	6	7 10	10	1.001	F ₅ :	Seed "23" plate
		18	5	1.006	F ₅ :	Faint
107.....	7	7 06	12	1.998	F ₉	Seed "23" plate
		16	5	2.005	F ₉	
143.....	Apr. 8	16 02	8	4.340	F ₉	
		07	2	4.343	F ₉	
157.....	9	16 02	5	5.340	F ₂	Wratten "M" plate
		10	10	5.345	F ₁	
165.....	25	16 02	5	5.241	F ₂	Seed "23" plate
		06	3	5.244	F ₂	
		11	6	5.247	F ₃	
173.....	27	14 48	1.5	1.823	F ₈	Bad seeing .
		50	1	1.824	F ₇	
		51	1	1.825	F ₈	
		52	2	1.826	F ₇	
182.....	28	14 04	5	2.792	G ₁	Seed "23" plate
		07	2	2.794	G ₂	
186b.....	July 9	11 10	1	4.908	F ₄	
		14	2	4.911	F ₄	
		18	5	4.914	F ₂	
193.....	10	10 45	6	0.525	F ₂	
		48	1	0.527	F ₃	
		52	4	0.530	F ₄	
218.....	30	9 26	8	4.371	F ₇	45°
		36	12	4.378	F ₅	
		44	3	4.383	F ₅ :	
219b.....	30	11 12	5	4.444	F ₅	45°
		19	8	4.449	F ₅	
232.....	31	9 02	5	5.354	F ₀	Seed "23" plate
		06	3	5.357	F ₁	
245.....	Aug. 1	9 28	9	1.006	F ₃	45°
		34	4	1.010	F ₂	
271.....	3	14 14	11	3.204	G ₀	45°
		23	8	3.210	G ₁	

* Luizet, *Annales de l'Université de Lyon*, Nouvelle Série, Fascicule 33, 1912.

MOUNT WILSON SOLAR OBSERVATORY
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THE DENSITIES OF VISUAL BINARY STARS¹

By E. ÖPIK

1. The density of a visual binary can be determined if the surface brightnesses of the components are known. Let R be the absolute radius; A , the apparent brightness; I , the surface brightness of the principal component (the corresponding values for the sun = 1); π , the parallax in seconds of arc; then

$$R = \frac{206265}{\pi} \sqrt{\frac{A}{I}}. \quad (1)$$

If M and M_1 are the masses ($\odot = 1$) of the components; a , the major axis; t , the period of revolution, we obtain the relation

$$M + M_1 = \frac{a^3}{t^2 \pi^3}. \quad (2)$$

Let δ be the density of the first component; then

$$M = \delta R^3, M + M_1 = \delta R^3 \left(1 + \frac{M_1}{M} \right),$$

and from (2)

$$\delta = \frac{a^3 \sqrt{\left(\frac{I}{A} \right)^3}}{t^2 \left(1 + \frac{M_1}{M} \right) \cdot 206265^3}. \quad (3)$$

Let m be the stellar (apparent) magnitude of the component; i , the surface brightness, expressed in stellar magnitudes ($i_\odot = 0.0$); then, assuming the stellar magnitude of the sun to be -26.60 , we obtain:

$$A = 10^{-0.4(m+26.6)} \quad (4)$$

$$I = 10^{-0.4i}. \quad (5)$$

After substituting these values in (3), it is easy to obtain the following formula:

$$\log \delta = \log \frac{a^3}{t^2} + 0.6(m-i) + 0.02 - \log \left(1 + \frac{M_1}{M} \right). \quad (6)$$

¹ *Publications of the Russian Astronomical Society*, No. 3, p. 49, 1915.

The density δ_1 of the other component can be computed if we in (6) instead of m , i , and $\frac{M_1}{M}$ substitute m_1 , i_1 and $\frac{M}{M_1}$.

2. The surface brightness depends upon the absolute temperature, and, consequently, upon the spectral type; such a dependence can be derived from the color-index, as has been done by H. N. Russell and Harlow Shapley.¹ I have preferred to use for this purpose the effective temperatures of 109 stars, as determined by Wilsing and Scheiner;² the mean temperatures for various spectral classes are given in Table I (the Harvard classification is introduced

TABLE I

Spectral Class	No. of Stars	T Effective Temperature	Spectral Class	No. of Stars	T Effective Temperature
Bo-B5.....	6	9030°	K.....	35	3970°
B8-A4.....	29	8880	M.....	8	2960
A5-F8.....	20	5780	Sun.....	(1)	5130
G.....	11	4450			

instead of Vogel's, used by the authors). If L and L_1 denote the wave-lengths of maximum spectral energy of two luminous sources, we obtain from Planck's formula for the relation of their surface brightnesses in the spectral region λ

$$\frac{I}{I_1} = \frac{e^{\frac{4.965L_1}{\lambda}} - 1}{e^{\frac{4.965L}{\lambda}} - 1}. \quad (7)$$

For "visual" light $\lambda_{\text{eff}} = 0.56\mu$, and assuming $\frac{4.965}{\lambda} = 8.87 = c$, it is easy to obtain for the difference (in stellar magnitude) of visual surface brightnesses of star and sun the equation

$$i = 2.5 \log \frac{e^{cL} - 1}{e^{cL_1} - 1}. \quad (8)$$

From Wien's formula, $LT = 2940$, L can be computed if T is known.

¹ *Astrophysical Journal*, 40, 417, 1914.

² *Astronomische Nachrichten*, 183, 97, 1909.

The equation (8) can be judged as true only when the *emissive powers* for star and sun are equal; that condition will be assumed in the following computations. It is probable that the order of magnitude of the radiating power for such dense and compact cosmical bodies does not differ much from unity in the majority of cases, but for stars of very low density and for nebulae exception must be made. In the case of the sun it is not difficult to confirm our assumption: it is sufficient to compare the solar constant of about 2 calories with the theoretical value corresponding to a temperature of about 6000° (with aid of the Stefan law).

The effective temperatures given by Wilsing and Scheiner are probably systematically too low, e.g., the temperature 5130° , found for the sun, corresponds to a value of the solar constant = 1.16 cal., which is smaller than the amount of radiation directly observable (at Mount Whitney, $1.6-1.7$ calories). The much more numerous and direct measures of the sun's spectral energy-distribution, made by Abbot and Fowle, led to a value of $L_{\odot} = 0.47\mu$ or $T_{\odot} = 6250^{\circ}$. We will assume this value; the observations of Wilsing and Scheiner we will consider as differential, and correct them so that the reduced temperature of the sun will equal 6250° .

In the Potsdam observations the intensity of radiation in a given spectral region of the star was compared with a terrestrial source of known energy-distribution or temperature; the star was directly compared with an electric lamp, the lamp with a furnace. In this manner, from the observations, the first part of equation (7) was obtained (I_1 and L_1 denote in this case the intensity of radiation and the spectral maximum of the furnace); that equation gives L if L_1 is known. Let us suppose that the assumed value of L_1 is in error; from (7) it is easy to obtain the differential formula

$$\Delta L = \frac{I_1}{I} \frac{e^{cL}}{e^{cL_1}} \Delta L_1, c = \frac{4.965}{\lambda}.$$

If cL and cL_1 are not too small (e.g. > 3), one can write

$$\frac{e^{cL_1}}{e^{cL}} = \frac{e^{cL_1} - 1}{e^{cL} - 1} = \frac{I}{I_1} \text{ and } \Delta L = \Delta L_1.$$

All values of L are modified by a nearly constant amount; that is approximately true for L not less than $0.3-0.4\mu$.

According to Wilsing and Scheiner $L_{\odot} = \frac{2940}{5130} \mu = 0.57\mu$; according to Abbot, 0.47μ ; thus we have $\Delta L = -0.10\mu$ = approximately ΔL_1 , which corresponds to an error of the absolute temperature of the electric furnace equal to 90° (at a temperature about 1600°). There can doubtless be other sources of error (or, if preferable, systematic difference), but we will for convenience assume the foregoing hypothesis; our only intention is to find a plausible formula for reducing the observations.

Let L' and L'_{\odot} be the maximum-energy wave-lengths corresponding to Wilsing and Scheiner's effective temperatures for star and sun, L and L_{\odot} , the reduced values; then, assuming I/I_1 to be correct, we obtain the equation

$$\frac{e^{cL'-1}}{e^{cL'_{\odot}-1}} = \frac{e^{cL-1}}{e^{cL_{\odot}-1}}, \left(c = \frac{4.965}{\lambda} \right) \quad (9)$$

from which L can be found if L , L_{\odot} , and L'_{\odot} are given. Theoretically for various λ the result must be the same if Planck's formula can be applied to stars and the observations are correct. The computation was made for $\lambda = 0.56\mu$, $c = 8.87$, $L_{\odot} = 0.47\mu$, $L'_{\odot} = 0.57\mu$, $L' = \frac{2940}{T'}$, where T' is the effective temperature; the result is given in Table II.

TABLE II

T' Effective Tempera- ture According to Wilsing and Scheiner	REDUCED VALUES		T' Effective Tempera- ture According to Wilsing and Scheiner	REDUCED VALUES	
	L	T		L	T
2500 ^o	1.000 μ	2700 ^o	6000 ^o	0.387 μ	7590 ^o
3000.....	0.886	3320	7000.....	0.314	9350
3500.....	0.746	3940	8000.....	0.258	11,400
4000.....	0.638	4600	9000.....	0.213	13,800
4500.....	0.554	5300	10,000.....	0.173	16,900
5000.....	0.488	6030	12,000.....	0.115	25,500
5500.....	0.434	6780	13,000.....	0.089	33,000

With aid of that table the reduced value of L for each star of Wilsing and Scheiner's list was found and the means for various spectral subdivisions were formed. The results are given in

Table III; in the third column L is the mean reduced wavelength of the spectral energy-maximum, the second and fourth columns give the number of stars n and the mean deviation Δ of L for one star; in the last column i denotes the surface brightness, computed with aid of equation (8) and the value of L from the third column. An inspection of the table leads to an interesting conclusion. The variation of temperature with spectral type goes on very irregularly; but it is not permissible to smooth out these

TABLE III

Spectral Class	n	L	Δ	$\left(\frac{T=2940}{L}\right)^*$	i Surface Brightness
Bo, B2, B5	5	0.196 μ	$\pm 0.07\mu$	15,000°	-3.0
B8	5	0.194	0.03	15,100	-3.0
A0	18	0.218	0.04	13,500	-2.8
A2	6	0.260	0.07	11,300	-2.2
A5	2	0.310	9480	-1.8
[A8p]	1	0.410	7160]	(-0.7)
F0	7	0.350	0.05	8390	-1.2
F5	5	0.422	0.04	6970	-0.6
F8	5	0.422	0.04	6970	-0.6
Go	6	0.548	0.07	5360	+0.7
G5	5	0.586	0.08	5010	+1.1
K0	28	0.614	0.08	4790	+1.3
K2	4	0.714	0.05	3930	+2.5
K5	3	0.877	0.07	3350	+3.9
Ma	8	0.895	0.03	3280	+4.1
Sun (Go)	0.470	6250	0.0

numbers, because the spectral classification is not quantitative, but merely qualitative, and it seems not to be correct to assume, for instance, that the spectral classes correspond to equal differences of temperature. The abrupt changes between some stellar classes seem to be real, e.g., between F8 and Go or Ko and K2. If we assume that various temperatures indicate different stages of stellar evolution in a cooling process, and that the fall of temperature goes on more or less uniformly with time, we are led to the conclusion that at some stages the variation of spectral qualities determining the type is very slow (as between F8 and Go, Ko and K2); at others, more rapid.

The sun can be placed between F8 and Go; its temperature is higher than the mean for the Go stars investigated.

3. To compute the density of a binary system the mass-ratio of the components must be known, which in but few cases is known with much precision; for our purposes, however, an approximate mass-ratio is sufficient. In cases when no reliable determinations of M_1/M were available, the masses were assumed to be equal if the difference of magnitudes of the components was $m_1 - m \leq 0^{\text{M}}8$. For the other cases the following process was used: for 11 stars¹ with known mass-ratio (determined generally by Struve, Auwers, Boss, and in one case each by Lewis and by Seeliger), the values of M_1/M were plotted as ordinates, while the abscissae were $m_1 - m$ (difference of visual magnitude of components); a smoothed curve was drawn, the points of which are represented in Table IV.

TABLE IV

Difference of Magnitude $m_1 - m$	Mass Ratio $M_1 : M$
$0^{\text{M}}0$	1.00
0.8.....	1.00
1.0.....	0.95
2.0.....	0.88
3.0.....	0.76
5.0.....	0.60
10.0.....	0.30

4. The entire data for computing the density are divided into two groups. In the first group are the binaries with $m_1 - m \leq 0^{\text{M}}8$; assuming the ratio $M_1/M = 1$, we can compute the mean logarithm of density for both components with aid of the modified equation (6):

$$\log \delta = \log \frac{a^3}{P} + 0.6 \left(\frac{m+m_1}{2} - i \right) - 0.28, \quad (10)$$

where m and m_1 are the visual magnitudes; i the surface brightness, from Table III, corresponding to the mean spectral type of the system.

For $m_1 - m > 0^{\text{M}}8$ in the majority of cases no data concerning the spectral type of the fainter component were available, and only the density of the brighter component was computed with aid of equation (6).

¹ The stars were: ϵ Hydrae, δ Herculis, α Can. Maj., η Cassiopeiae, α Can. Min., ξ Cancri, ξ Ursae Maj., α Centauri, γ Ophiuchi, ξ Boötis, γ Virginis.

It is evident that the stellar magnitude of star and sun must be referred to the same photometric system; the magnitude of the sun = -26.60 corresponds to the Potsdam system, and for this reason the magnitudes of the components were so chosen that the equations

$$10^{-0.4m_0} = 10^{-0.4m_1} + 10^{-0.4m_2} \quad (11)$$

and

$$m_1 - m_2 = \Delta \quad (12)$$

were fulfilled, where m_0 is the stellar magnitude of the whole binary system according to the Potsdam scale; Δ , the difference of magnitude of the components.

In Tables V and VI are given the results of the computation; in the sixth column the parentheses indicate that the corresponding mass-ratio is found with aid of the empirical relation given in Table IV.

TABLE V

STAR	m_0 P. D.	SPEC- TRAL TYPE	STELLAR MAGNITUDE OF COMPONENT		MASS- RATIO $M_1 : M$	$\log \frac{a_1}{P}$	DENSITY	
			m_1	m_2			$\log \delta$	$\frac{\delta}{(\odot = 1)}$
α_1 Centauri.....	G	0.3	0.96	9.924-10	-0.60	0.25
α_2 Centauri.....	K5	2.1	-1.44	0.036
Stars with $m_1 - m_2 \leq 0.8$; mean density of the system								
γ Virginis.....	2.7	F	3.5	3.5	1.0	7.227-10	-0.23	0.59
δ Equeulei.....	4.7	F5	5.2	5.7	(1.0)	6.682	0.03	1.07
ι_3 Ceti.....	5.8	F	6.2	6.9	(1.0)	6.226	0.60	4.00
κ Pegasi.....	4.3	F5	4.7	5.5	(1.0)	6.274	-0.59	0.26
δ Sagittarii.....	2.6	A2	3.2	3.4	(1.0)	6.592	-0.39	0.41
β 612.....	5.8	A	6.5	6.6	(1.0)	5.424	0.75	5.6
η Argus.....	5.3	F8	5.8	6.4	(1.0)	6.621	0.36	2.3
δ_2 Ceti.....	5.6	K	6.3	6.4	(1.0)	6.700	-0.55	0.28
α_2 Comae.....	4.6	F5	5.4	5.4	(1.0)	6.604	-0.08	0.83
β Delphini.....	4.0	F5	4.5	4.9	(1.0)	6.312	-0.79	0.16
α_2 Persei.....	5.7	F	6.1	6.9	(1.0)	4.500	-1.16	0.069
η Coronae Bor.....	5.2	G	5.7	6.2	(1.0)	6.611	-0.52	0.30
ξ Scorpii.....	4.2	F8	4.8	5.0	(1.0)	6.239	-0.74	0.18
Σ 2173.....	5.5	G	6.0	6.4	(1.0)	6.845	-0.14	0.72
δ Sextantis.....	5.3	A2	5.8	6.1	(1.0)	4.956	-0.43	0.37
γ Centauri.....	A	3.2	3.2	(1.0)	6.139	-0.54	0.29
ϕ Ursae Maj.....	4.7	A2	5.2	5.8	(1.0)	4.517	-1.14	0.072
ω Leonis.....	5.6	G	6.0	6.8	(1.0)	5.704	-1.16	0.069
γ Coronae Austr.....	F8	5.0	5.0	(1.0)	6.799	-0.12	0.76
Σ 2.....	6.3	A	6.9	7.2	(1.0)	4.780	0.41	2.6
ξ Cancri.....	4.8	F	5.2	5.9	1.0	6.258	0.03	1.07

Only in one case, for α Centauri, was it possible to obtain separate densities for both components, because in this case their individual spectral types are known.

In forming the means it is preferable to use the logarithms of δ instead of the densities themselves, because the data on which our computations are based are logarithms (m , m_1 , and i), and the

TABLE VI
STARS WITH $m_1 - m > 0.8$

STAR	m_0	SPEC- TRAL TYPE	m	m_1	$M_1 : M$	$\log \frac{m_1}{m}$	DENSITY OF THE BRIGHTER COMPONENT	
							$\log \delta$	δ ($\odot = 1$)
ϵ Hydrea.....	3.6	F8	3.7	5.7	0.90	5.748-10	-1.93	0.012
β Herculis.....	3.2	G	3.2	6.6	0.43	7.326	-1.31	0.049
Sirius.....	-1.4	A	-1.4	10.0	0.42	9.252	-0.04	0.91
η Cassiopeiae.....	3.7	F	3.7	8.0	0.76	8.053	0.77	5.9
Procyon.....	0.8	F5	0.8	9.6	0.26	8.915	-0.32	0.48
ξ Ursae Maj.....	3.9	G	4.2	5.1	1.0	7.640	-0.54	0.29
70 Ophiuchi.....	4.2	K	4.3	6.3	0.82	8.082	-0.36	0.44
ξ Boötis.....	4.8	K5p	4.9	6.8	0.87	7.750	-1.90	0.013
85 Pegasi.....	6.0	G	6.0	11.0	0.60	6.856	-0.14	0.72
β 416.....	5.9	K5	6.0	8.0	(0.80)	7.534	-1.47	0.034
A Cassiopeiae.....	4.7	A2	4.8	7.3	(0.81)	5.907	-0.13	0.74
τ Cygni.....	4.0	F	4.0	10.0	(0.53)	6.744	-0.30	0.50
99 Herculis.....	5.3	F8	5.3	11.0	(0.55)	6.701	0.07	1.17
Ω 235.....	5.7	F	6.0	7.3	(1.0)	6.117	0.16	1.45
γ Coronae Bor.....	4.0	A	4.1	6.9	(0.9)	5.881	-0.24	0.58
Σ 3062.....	6.1	F	6.4	7.5	(1.0)	6.369	0.65	4.5
25 Can. Venat.....	5.0	F	5.1	8.6	(0.72)	5.629	-0.81	0.16

deviations caused by errors of observation, are logarithmic deviations; if converted into numbers, the positive deviations would be greater than the negative for equal errors of observation, and the means would be greater than the true one.

The principal source of error is probably the uncertainty in spectral classification; an error amounting to one spectral class will alter the result some 4 times. In the foregoing tables are four exceptionally high values of δ : 4.0, 5.6, 5.9 (η Cassiopeiae), 4.5. Such densities are very improbable; they correspond to 5.6-8.2 times the density of water, and are probably due to errors in the assumed surface brightness (spectral type); in the case of η Cassiopeiae the elements of the orbit are too uncertain.

As appears from the tables, the densities cover a wide range, but there are a great number of binaries approaching the solar density. From this point of view the visual binaries differ substantially from the Algol variables, which give generally very low densities; it is probable that the visual systems approach much more the mean conditions of the stellar universe, and the study of their densities, masses, etc., can give us a fairly approximate idea of the constitution of the great number of stars with low or moderate luminosity, which form the prevailing mass in the galactic system. The eclipsing variables, however, are known to be exceptions to the general rule; their luminosity is much higher than the average, the stars of early spectral type prevail among them, and numerically they represent but a small portion of all the stars.

In Table VII the number of binaries is arranged according to the density; 10 binaries, or 25 per cent of the total number, have densities greater than the sun. The numbers of Table VII are plotted on Fig. 1 as rectangles; the line represents the hypothetical "density-curve," or the number of stars with a given $\log \delta$; the maximum of the curve is at $\log \delta = -0.23$ or $\delta = 0.59$.

TABLE VII

Limits of $\log \delta$	No. of Binaries	Percentage	Limits of $\log \delta$	No. of Binaries	Percentage
-2.0 to -1.5...	2	5	-0.5 to 0.0...	13	32
-1.5 -1.0...	6	15	0.0 0.5...	6	15
-1.0 -0.5...	9	23	0.5 1.0...	4	10

In Table VIII are given the mean values of $\log \delta$ and the corresponding δ for different spectral classes. There is a marked decrease of density in passing from the early to the later types.

TABLE VIII

Spectral Type	No. of Stars	Mean $\log \delta$	Corresponding δ
A0-A5.....	9	-0.19	0.65
F0-F8.....	19	-0.23	0.59
G.....	7	-0.63	0.23
K, K5.....	5	-1.14	0.072
Total.....	40	-0.407	0.30

The foregoing results are only rough, because the data are strongly affected by observational errors. Assuming a probable error $= o^{M1}$ for the visual magnitude and one-fourth spectral class for the spectral data, which corresponds to an error in i equal to o^{M25} , we obtain the probable error of a density-logarithm $= \pm o.61 \sqrt{o.1^2 + o.25^2} = \pm o.16$; that corresponds to 45 per cent of the value to be determined.

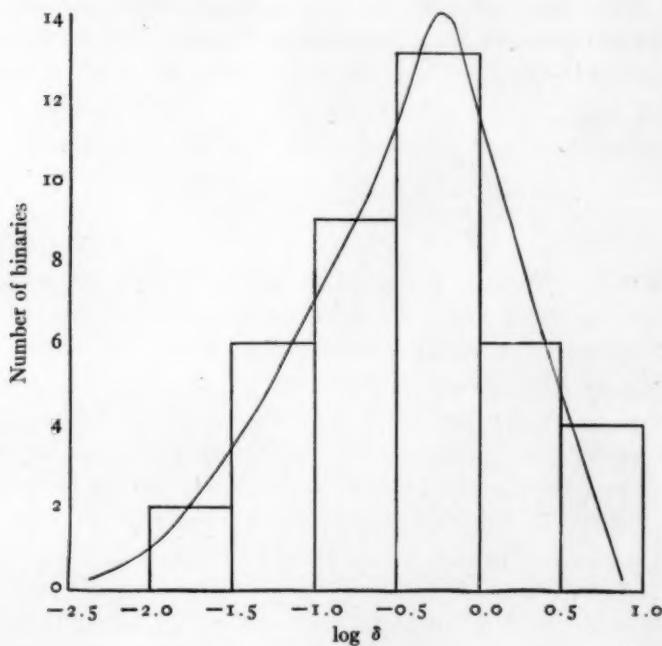


FIG. 1

It is desirable that more accurate determinations of magnitude and spectral type of visual binaries with known orbital elements should be made; instead of spectral type it is sufficient to determine the color-index. When such data for each component of the binary become available, it will be possible to obtain separately the densities of the components, and a very important and interesting region of stellar statistics will be opened.

Among the binaries with known orbital elements there is one which is not included in the preceding discussion, α_2 Eridani; the

components are $9^m 2$ and $10^m 9$ respectively, and, according to Adams and Pease,¹ of the spectral type Ao; the parallax is $0''.17$, and the absolute luminosity of the components must be $\frac{1}{100}$ and $\frac{1}{800}$, respectively, of the sun's—values exceptionally small for A-type stars; the density according to equation (6) ($M_1/M=1$) would be 25,000. This impossible result indicates that in this case our assumptions are wrong; the only possible explanation is that, however high the temperature, the surface brightness or the radiating power is very low; probably α_2 Eridani is a pair of very rarified nebulae.

MOSCOW, RUSSIA
May 1916

¹ *Publications of the Astronomical Society of the Pacific*, 26, 258, 1914.

PRELIMINARY OBSERVATIONS OF THE SPECTRA OF CALCIUM AND IRON WHEN PRODUCED BY CATHODO-LUMINESCENCE¹

By ARTHUR S. KING AND EDNA CARTER

The excitation of a vapor to luminescence by means of a stream of cathode rays directed into it furnishes a source different from those in which the radiation is produced by high temperature or by the conduction of a current through the vapor. The purpose of the present investigation was to study the peculiarities of the spectrum given by the cathode-ray excitation for certain elements having groups of lines known to show different behavior in the ordinary sources.

The possibility of producing spectra in this way was shown by Hertz,² who obtained the stronger mercury lines, and by Lewis,³ who examined the spectra of several of the more easily vaporized metals, the vapor being produced by heating the substance in a hard glass tube by means of a flame and directing a stream of cathode rays into the tube. In each case a spectrum of what seemed to be the more fundamental lines of the element was observed. The present writers have been aided by valuable suggestions from Professor Lewis, obtained during a discussion of the problem.

The heating effects regularly observed in X-ray tubes since the introduction of tungsten targets made it appear feasible to work with some of the more refractory metals by heating them with a concentrated stream of cathode particles. When brought to luminescence the vapor in the cathode stream could then be observed at a sufficient distance from the heated surface to avoid the direct effects of high temperature.

The first apparatus constructed was a modified X-ray bulb, with concave cathode and disk anode in the usual positions. The

¹ Contributions from the Mount Wilson Solar Observatory, No. 125.

² Wiedemanns Annalen, 19, 809, 1883.

³ Astrophysical Journal, 16, 31, 1902.

cathode stream was directed vertically downward upon an anti-cathode consisting of a tungsten dish containing metallic calcium, supported on the end of a silica tube passing into the bulb through the end opposite the cathode. When the bulb was pumped out to a vacuum sufficient to give in air a parallel spark 10–15 cm long, the calcium was readily vaporized and the pencil of cathode rays above the dish showed, when observed visually with a small spectro-

scope, a spectrum containing a considerable number of lines. The lines H, K, and $\lambda 4227$ were photographed with a 1-meter concave grating.

A more substantial apparatus was then made from a bell-jar supported on a thick glass plate, as shown in Fig. 1. The cathode C is 25 mm in diameter, concave to such a degree as to focus the rays at 9.5 cm when the spark crosses a parallel air-gap of 15 cm. It is screwed to an aluminum rod passing through the top of the bell-jar, while a flaring glass tube around the electrode protects the wall of the jar from the cathode dis-

charge. The anode A is a vertical aluminum rod 5 mm in diameter, passing through the glass base and inclosed by a glass tube, the inner side of which is cut away for a length of 4 cm below the end of the rod in order to expose the anode and at the same time protect the wall of the jar. The anti-cathode D is supported by a silica tube of 15 mm bore, held vertically by being set in a glass plate resting on the base plate. This tube holds at its top either a tungsten dish containing calcium or a shallow iron dish with iron filings. The dish in each case is at the focus of the cathode and is several centimeters away from the direct discharge between anode and cathode. As the experiments of Lewis with metals and with nitrogen¹ showed clearly the possibility of bringing gases outside the path of the conduction

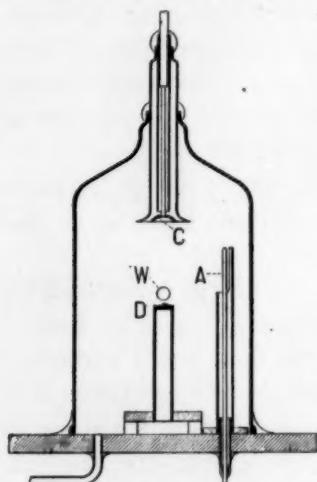


FIG. 1

¹ *Astrophysical Journal*, 17, 258, 1903.

current to luminescence by cathode rays, no special examination of this point was made during the present investigation. A tube through the base-plate for the pump connection and a side tube inserted at the opening *W* in the bell-jar and closed by a quartz window completed the fittings. The joints in the apparatus were cemented either with sealing-wax or with an odorless black wax of nearly the same consistency. The discharge was obtained from a large induction coil driven by an electrolytic interrupter of the Simon type, which, when the vacuum was sufficiently high, gave little inverse current.

When calcium was vaporized by the cathode stream, the vapor immediately absorbed the residue of air—an effect noted by Saunders¹—which made the vacuum so high that the discharge would not pass until a little air was admitted. The lumps of metallic calcium were found to be almost consumed by the discharge, which could not be closely watched on account of an opaque deposit that blackened all parts of the chamber except the window toward the spectrograph. The iron was raised only to a white heat, the filings not being fully fused. However, sufficient iron vapor was given off in the high vacuum to enable the cathode rays to yield a spectrum. In focusing the light on the spectrograph, the image of the metal anti-cathode was kept well off the slit, the light photographed coming from the cathode stream about a centimeter above the heated metal.

As the experiments will be discontinued for a short time, pending the installation of apparatus which may be expected to give much stronger effects, the following results are presented in the belief that they are typical of what such a discharge will yield. The spectra were photographed in the first order of a 1-meter concave grating, the scale being 1 mm = 17 Å. The metallic lines were very sharp and the measurements for identification usually agreed with the published values within 0.02 Å. Two reasonably strong spectrograms were obtained for calcium, in which the 26 lines listed in Table I were identified. These were supported by three weaker photographs giving the same effects for such lines as appeared. Three photographs of the iron spectrum were made, which were

¹ *Ibid.*, 40, 377, 1914.

also concordant in the effects shown. While the 42 iron lines obtained are a small fraction from the rich region between $\lambda 3400$ and $\lambda 4400$, they show the class of lines which appear under these conditions.

RESULTS

Calcium.—Table I gives a comparison of the relative intensities of the lines in the luminescence spectrum and in that of the arc in air. An arc spectrum of very short exposure on the same kind of film was used in order to obtain a general intensity comparable with that of the spectrum excited by the cathode discharge. In the fourth column is entered the symbol used by Saunders¹ to denote the series to which the line belongs.

TABLE I
LUMINESCENCE SPECTRUM OF CALCIUM

λ (Exner and Haschek)	Arc in Air	Lumines- cence Spectrum	Series	λ (Exner and Haschek)	Arc in Air	Lumines- cence Spectrum	Series
3170.50...	1	1	P ₁	4283.20...	15	p
3181.43...	trace	trace	P ₁	4289.50...	15	T
3350.25...	2	T ₁	4299.18...	10	T
3361.95...	4	T ₁	4302.70...	20	T
3624.19...	4	T ₁	4307.90...	15	p
3630.87...	8	2	T ₁	4318.80...	15	T
3644.53...	15	4	T ₂	4355.50...	2	7	SL ₃
3706.18...	1	4	P ₂	4425.60...	20	3	T ₁
3737.06...	2	8	P ₂	4435.17...	30	8	T ₁
3933.81...	80	40	PH	4435.88...	8	1	T ₁
3949.10...	1	T ₂	4455.00...	40	15	T ₁
3957.22...	4	trace	T ₂	4456.10...	10	1	T ₁
3968.63...	60	30	PH	4527.35...	3	8	SL ₂
3973.91...	6	2	T ₂	4578.88...	3	t
4093.00...	1	t	4581.77...	6	1	t
4095.30...	2	t	4586.22...	8	2	t
4098.9...	4	t	4685.35...	trace	5
4108.60...	trace	4	SL ₂	4878.38...	3	4	SL ₃
4226.90...	100R	400	SL ₁	5041.83...	1	1	SL ₂
4240.61...	1	4	SL ₂				

In the luminescence spectrum, the lines H, K, and $\lambda 4227$ are much the strongest, and perhaps the most striking feature is the phenomenal intensity, combined with sharpness, of $\lambda 4227$. In the arc and furnace, even in vacuum, this line widens and reverses.

¹ *Astrophysical Journal*, 32, 154, 1910.

more strongly than any other line in the calcium spectrum. Its intensity, so far as this can be based on widening, has seemed a reliable indication of the vapor-density of the source. When excited by cathode rays in the present experiments, a strong central maximum is present with apparently no tendency toward reversal. This is illustrated by the fact that while the grating used does not usually give perceptible ghosts, even of very strong lines, the first-order ghosts of $\lambda 4227$ are now of about the same intensity as the H line. Apparently we have here a low vapor-density, resulting in an extremely narrow line, combined with other conditions highly favorable for the production of $\lambda 4227$. The strength of $\lambda 4227$ is the only resemblance to the furnace spectrum. The luminescence spectrum brings out strongly H and K and the enhanced pair $\lambda 3706$ and $\lambda 3737$. The latter have not been obtained in the furnace, and H and K are weak in the furnace as compared with the arc. $\lambda 3159.01$, belonging to the same series as the first two lines in Table I, is probably present, but blends with the diffuse head of a nitrogen band.

Another noteworthy feature of the luminescence spectrum is the weakness of the group of six lines near $\lambda 4300$, of which only the strongest, $\lambda 4302.70$, was photographed. This group is one of the least variable in the usual sources, including the furnace at different temperatures.

Table I shows further that the series triplets are uniformly weak in the cathode discharge as compared with the arc, while the members of the "single-line" (SL) series of Saunders are much stronger than in the arc. Some of these, such as $\lambda 4108.60$, are diffuse in the arc in air, and are rendered more distinct by any of the vacuum sources, but this does not account for the high relative intensity obtained here. Saunders¹ has recently placed $\lambda 4227$ also in a single-line series, of which the other members are in the ultraviolet.

If a general feature of this discharge is an excitation of the more fundamental vibrations, as the experiments of Lewis seemed to indicate, the several series, consisting of single lines, of pairs, and of triplets, respectively, are of increasing complexity in their

¹ *Ibid.*, 43, 234, 1916.

vibrations, since they are arranged in this order in the facility with which they are given by the cathode luminescence.

There is no evidence that new lines are produced by the cathode discharge and, with the exception of certain nitrogen bands referred to later on, all lines in the luminescence spectrum have been identified with known lines of calcium.

TABLE II
LUMINESCENCE SPECTRUM OF IRON

\AA (Rowland)	Int.	\AA (Rowland)	Int.
3440.762		3767.341	trace
3441.155	4	3813.100	trace
3570.273	3	3815.987	1
3581.349	10	3820.586	4
3609.008	1	3824.591	2
3618.919	1	3826.027	2
3631.605	1	3834.304	1
3647.988	1	3840.580	trace
3680.069	trace	3856.524	3
3687.610	trace	3860.055	15
3705.708	1	3878.720	2
3720.084	20	3886.434	5
3722.729	1	3920.410	1
3727.778	trace	3923.054	1
3735.014	7	3928.075	1
3737.281	12	3930.450	1
3745.717	8	4045.975	1
3748.408	4	4308.081	trace
3749.631	4	4325.939	trace
3758.375	2	4383.720	2
3763.945	1	4404.927	trace

Iron.—The list of iron lines identified, with their estimated intensities, is given in Table II. Their general characteristic is that they are the strongest lines in the flame, furnace, arc, and spark spectra. Hemsalech and De Watteville¹ observed all of the lines listed in Table II, and a few others in the oxyhydrogen flame. Although considerable differences in relative intensity appear between the flame and the cathode excitation, the latter seeming stronger in the ultra-violet, we cannot tell how far differences in contrast and in the treatment of plates may account for the variations observed. The strongest lines of the luminescence spectrum

¹ *Comptes Rendus*, 146, 962, 1908.

are also the strongest in the flame, but many very faint luminescence lines are given as of considerable strength in the flame.

The furnace material available, part of which has not been published, shows that the lines of Table II appear at low temperatures, below 2000° C., and reverse readily at temperatures above 2300°. They would be placed uniformly in Class II of the furnace classification. In the arc and the condensed spark these are among the strongest lines, reversing readily with high vapor-density. The strongest lines of the luminescence spectrum, $\lambda\lambda$ 3581, 3720, 3737, 3860, are of great strength in all of the usual sources. The lines most characteristic of the furnace (Class I) and the enhanced lines of the spark are, however, absent from the luminescence spectrum. While this may be due in some degree to the general weakness of the latter in the photographs obtained, it can at least be said that such lines do not dominate the spectrum, as they do in the sources favorable to them. A fair statement would be that the lines thus far obtained are those of an under-exposed arc spectrum of iron. When more lines are photographed we may expect, among various groups, differences similar to those occurring in the calcium spectrum.

Band spectrum of nitrogen.—The following nitrogen bands appeared on the photographs of the luminescence spectrum:

Positive Pole Bands	Negative Pole Bands
3158.9	3581.5
3371.2	3914.4
3536.5	4236.3
3576.9	4278.0
3755.2	4708.6
3804.9	
4059.0	

While no attempt was made to locate closely the region of production, the appearance of the bands under these conditions is in harmony with the conclusions of Lewis,¹ that the negative pole bands are produced by the impact of cathode rays, while the positive pole bands are given in all parts of the chamber. The concentration of the cathode discharge should favor the negative bands, and

¹ *Astrophysical Journal*, 17, 258, 1903.

this is borne out by the fact that they are much the stronger. The heads of the positive pole bands appear as diffuse lines, while the negative bands λ 3914 and λ 4278 are strong enough to show considerable structure. These bands present the curious appearance of fading out a short distance from the head and then continuing again. This was observed by Deslandres¹ to be peculiar to low pressures, and to result from the suppression under these conditions of one of the series of lines composing the band, only a few of the first members of this series appearing, while at atmospheric pressure the full structure is present.

SUMMARY

1. A method, applicable to a wide variety of elements, has been developed for the production of metallic spectra through the luminescence excited by cathode rays.
2. The spectrum of calcium in this discharge consists of lines present in the arc, but differing in relative intensities from the arc and other sources. The lines of the several single-line series are strongest, followed by the lines occurring in pairs, while the triplet series are relatively faint.
3. The iron lines thus far obtained belong to what appears to be the fundamental group in the iron spectrum, strong in all of the usual sources. Lines characteristic of the low-temperature sources and of the spark are absent.
4. Bands of the positive and negative pole spectra of nitrogen have been photographed, the negative bands having high intensity.

MOUNT WILSON SOLAR OBSERVATORY
October 9, 1916

¹ *Comptes Rendus*, 139, 1174, 1904.

OBSERVATIONAL EVIDENCE THAT THE RELATIVE POSITIONS OF FRAUNHOFER LINES ARE NOT SYSTEMATICALLY AFFECTED BY ANOMALOUS DISPERSION¹

BY CHARLES E. ST. JOHN

I. INTRODUCTION

The suggested mutual influence of Fraunhofer lines, a quasi-repulsion between closely adjacent lines, offers a ready means of testing the theory of anomalous dispersion in the solar atmosphere. Professor Julius² developed the deduction and applied it to the displacements between the center and limb edges of the penumbras of sun-spots given in my paper on "Radial Motion in Sun-Spots."³ He found what he considered positive evidence of the effect. I rediscussed⁴ the data from this point of view, including a large number of omitted lines fulfilling the conditions of selection, and calling attention to errors involved in the method used by Julius which introduced systematic differences of the sign required by the hypothesis. Of this discussion Julius says:

A few months later St. John published the elaborate article in which he criticized my treatment of his observations very severely, in several respects justly. I had not overcome the difficulties of handling rightly the Mount Wilson data, nor had I entirely avoided bias. St. John made certain alterations in the method of grouping and comparing the measured displacements, added a number of omitted and of new cases, and thus reached the conclusion that there was no indication at all of a mutual influence.

Although I am satisfied by St. John's improved discussion of the data that, in the Evershed effect, mutual influence is not so conspicuous as my defective treatment of those measurements had made it appear, I still believe that future research will bring it to light.⁵

The magnitude of the effect in this case Julius now says can scarcely be expected, under the most favorable conditions, to exceed

¹ *Contributions from the Mount Wilson Solar Observatory*, No. 123.

² *Astrophysical Journal*, 40, 1, 1914.

³ *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, 37, 327, 1913.

⁴ *Mt. Wilson Contr.*, No. 93; *Astrophysical Journal*, 41, 28, 1915.

⁵ *Astrophysical Journal*, 43, 49-50, 1916.

0.002 Å, a quantity so near the limit of precision for such observations that its definitive establishment must prove a matter of extreme difficulty, a quantity so small that the question becomes one of theoretical rather than practical interest to the astrophysicist.

Albrecht¹ found by comparing the Rowland and International wave-lengths of iron lines that the violet and red components of close solar pairs showed, relative to the general displacement, specific displacements to the violet and red of 0.007 and 0.005 Å, respectively, for a mean separation of 0.22 Å from the companion line. These results he interpreted as effects of anomalous refraction, an interpretation approved by Julius, who accepts the data as definitely establishing the existence of mutual influence between the lines in the general solar spectrum.²

II. SUN-ARC DISPLACEMENTS

Quantities of the order given by Albrecht's investigations are well within the range of present instruments and methods. The displacements of the solar lines in sun-arc comparisons furnish a direct and suitable means of investigating the question, as a difference of 0.012 Å between the displacements of lines with violet and red companions is greater than the average value of the sun-arc displacements, quantities determinable with high accuracy. Such data, center of sun *minus* center of arc, have been accumulating at this Observatory for three or four years. The early observations were made with the 75-foot spectrograph. The length of the plates (90 cm) and the scale (1 mm = 0.7 Å) are such that a range of spectrum of 600 Å is covered by a single exposure. This has insured such interlocking that the results may be considered free from serious relative errors. Recently a series of plates was begun under still more refined conditions with the remodeled 30-foot spectrograph of the 60-foot tower telescope. Measurements now in progress upon these plates are confirming the absolute values previously obtained for the groups of stable arc lines.

¹ *Astrophysical Journal*, 41, 333, 1915.

² *Ibid.*, 43, 53, 1916.

In the Mount Wilson classification of iron lines,¹ those whose wave-lengths are independent of arc conditions at constant pressure are distributed among groups *a*, *b*, and *c*₄; to groups *c*₅, *d*, and *e* are assigned the lines subject to pole-effect,² whose wave-lengths therefore vary with the current-strength and the part of the arc used.

A. *Groups a, b, and c*₄.—In the present situation as to solar observations, conclusions based upon the relative behavior, in the sun and arc, of lines belonging to groups *a*, *b*, and *c*₄ carry the maximum weight. Of these groups 211 lines, sufficiently separated from neighboring lines for measurement of precision and free from known blends, have been examined for sun-arc displacements.

An investigation entitled "The Accuracy Obtainable in the Measured Separation of Close Solar Lines; Systematic Errors in the Rowland Table for Such Lines,"³ has recently been concluded. This has furnished the means for determining the minimum separation between lines in the solar spectrum, consistent with their accurate measurement upon spectrograms taken with the instrument employed in obtaining these sun-arc displacements.

The data relative to all lines of these groups having close companions are given in Tables I and II. To identify the lines common to Albrecht's list and mine, his values are shown in the last column. The first section of each table contains measurements free from known sources of error. The measurements in the second section are of little or no weight, the lines themselves being blends or the adjacent lines too near.

The 56 lines having companions to the violet, mean separation 0.275 Å, and the 29 having companions to the red, mean separation 0.320 Å, show displacements of +0.0036 and +0.0038 Å, respectively, in agreement with the mean of 0.0038 Å from the 211 lines, instead of approximately +0.009 Å and -0.003 Å, values required to harmonize with the observations of Albrecht,

¹ *Transactions of the International Union for Co-operation in Solar Research*, 4, 74, 1913.

² St. John and Babcock, "A Study of the Pole-Effect in the Iron Arc," *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, 42, 231, 1915.

³ St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

if the deviations found by him represent actual shifts of 0.005 and 0.007 to the red and violet relative to the general displacement of +0.004 Å. The behavior of these 85 lines with closely adjacent companions, lines of high quality in both solar and arc spectra,

TABLE I
RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF IRON, GROUPS *a*, *b*, AND *c4*
COMPANIONS TOWARD THE VIOLET IN SOLAR SPECTRUM

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht
3964.663...	2: 3	.247	+ 0.001	4443.365...	1:3	.360	+ 0.003
3969.413...	6:10	.527	+ .013	- 0.004	4531.327...	2:5	.204	+ .004	- 0.002
3977.009...	3: 2	.170	+ .003	4556.306...	3:4	.243	+ .004	+ .002
4000.611...	2: 2	.208	+ .002	4592.840...	2:4	.133	+ .004	- .010
4007.814...	1: 3	.212	+ .007	4741.718...	1:3	.450	+ .001
4007.429...	5: 3	.290	+ .007	4787.003...	3:2	.276	+ .003	- .012
4087.253...	1: 3	.391	+ .004	4789.849...	2:3	.321	+ .002	- .013
4123.907...	1: 5	.243	+ .006	4939.868...	2:4	.452	+ .004	- .002
4133.062...	1: 4	.109	+ .005	5041.255...	3:4	.186	+ .000	- .002
4134.840...	3: 5	.251	+ .004	+ .006	5041.930...	2:4	.141	+ .012	+ .001
4137.156...	4: 6	.478	+ .003	5051.825...	1:4	.142	+ .006
4146.225...	1: 3	.311	+ .000	5079.409...	3:4	.251	+ .003	- .006
4147.836...	2: 4	.334	+ .000	+ .002	5098.885...	1:3	.134	+ .002	- .005
4149.533...	2: 4	.173	- .001	5107.823...	4:4	.204	+ .006	- .004
4152.343...	2: 3	.235	+ .006	5143.111...	2:3	.153	+ .004
4156.970...	1: 3	.139	+ .003	5227.362...	3:5	.319	+ .002	- .003
4172.923...	2: 4	.120	+ .003	5250.817...	2:3	.432	+ .003
4174.095...	3: 3	.385	+ .008	5328.606...	2:2	.181	+ .006
4191.843...	6: 3	.248	+ .006	+ .003	5333.089...	1:4	.247	+ .006	+ .005
4202.198...	1: 5	.329	+ .006	5305.590...	5:3	.537	+ .002	+ .003
4220.509...	1: 3	.297	- .002	5405.959...	1:6	.435	+ .006
4242.897...	2: 2	.131	- .004	5429.911...	1:6	.194	+ .007
4282.565...	2: 5	.438	+ .004	5447.130...	2:6	.333	+ .004	+ .003
4320.542...	2: 1	.105	+ .002	6137.210...	8:3	.381	+ .006
4321.630...	1: 2	.255	+ .002	+ .007	6191.779...	6:9	.386	+ .005	- 0.004
4325.262...	3: 4	.124	+ .006	- .007	6462.963...	5:3	.181	+ 0.008
4369.041...	1: 4	.373	+ .003	Mean, 56 lines.....275	+ 0.0036
4427.482...	2: 5	.210	+ .004	Mean, 23 lines, Albrecht.....283	- 0.0018
4430.785...	1: 3	.420	+ .001					
4435.321...	5: 2	.192	+ 0.005					

SECTION B. MEASUREMENTS WITHOUT WEIGHT

4394.301...	2: 5	.097	- 0.013	- 0.005	5167.678...	15:5	.181	+ 0.019	- 0.016
4407.871...	3: 4	.061	- .027	+ 0.011	6254.456...	1:3	.074	- 0.014	+ 0.017
4547.192...	1: 2	.091	+ .003					
4552.725...	2: 1	.093	+ .006					
4633.100...	1: 4	.109	+ 0.001	Mean, 7 lines.....101	- 0.0036

fails to show the large differences found by Albrecht, but, on the other hand, furnishes strong observational evidence that the sun-arc shifts for such lines are the same as for isolated lines and are therefore not measurably affected by mutual influence.

B. Groups *c5*, *d*, and *e*.—Measurements upon iron lines of groups *c5*, *d*, and *e* are so easily affected by conditions in the arc, that con-

sistent results can be expected only when these conditions are constant. In obtaining the sun-arc displacements at Mount Wilson, the Pfund arc used had a length of 6 mm and was fed by a current of 6 amperes from a 110-volt storage battery. The slit was normal to the axis and in the equatorial zone of a two-and-one-half-fold enlarged image produced by an achromatic lens. Under

TABLE II
RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF IRON, GROUPS *a*, *b*, AND *c4*
COMPANIONS TOWARD THE RED IN THE SOLAR SPECTRUM
SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht
4000.403...	2:2	.218	0.000	4454.552...	5:3	.401	+0.003	+0.003
4030.339...	5:2	.307	-0.001	4461.818...	3:4	.347	+0.004	+0.005
4067.139...	3:5	.290	+	.011	4466.737...	1:5	.375	+0.008
4082.264...	3:2	.325	-0.003	4489.911...	3:4	.342	+0.006	+0.004
4096.129...	2:3	.133	+	.003	4610.468...	1:3	.243	+0.004
4098.335...	4:5	.354	0.000	5079.921...	1:4	.223	+0.003
4136.678...	6:4	.478	+	.005	5107.619...	4:4	.204	+0.004	+0.005
4140.089...	3:6	.469	+	.006	5131.642...	1:3	.300	+0.005
4142.025...	2:4	.305	+	.001	5105.113...	2:4	.534	+0.004	+0.006
4171.068...	4:4	.145	+	.006	5328.230...	2:8	.270	+0.011	-0.007
4175.082...	1:4	.210	-0.001	5341.313...	1:7	.124	+0.010
4224.337...	3:4	.336	+	.005	5307.344...	1:7	.478	+0.008
4226.584...	20:2	.320	+	.006	6136.829...	3:8	.381	+0.002	+0.004
4267.122...	2:3	.421	-0.006	Mean, 29 lines.....320	+0.0038
4337.210...	3:5	.509	+	.004	+0.002	Mean, 8 lines, Albrecht.....
4351.711...	5:2	.219	+0.004375	+0.0035

SECTION B. MEASUREMENTS WITHOUT WEIGHT

3907.547...	3:4	.091	+0.032	+0.001	4476.185	3:4	.068	+0.025	+0.008	
4204.101...	4:3	.062	-0.011	+	.048	5012.252...	1:4	.083	+0.017	-0.002
4310.494...	3:4	.067	+	.002	+0.027	5169.069...	4:3	.151	+0.015
4245.422...	2:4	.098	+	.000	5470.500...	3:1	.278	0.000	+0.011
4307.749...	2:5	.090	+	.005	
4391.123...	1:2	.069	+0.003	Mean, 29 lines.....105	+0.0084	

the constant conditions, the displacements for lines of groups *c5*, *d*, and *e*, relative to each other, are probably free from serious error, and comparison between the behavior of lines with companions, and that of isolated lines, can yield results of weight.

The Mount Wilson data include 125 lines of the related groups *c5* and *d*, and 34 lines of group *e*. The mean sun-arc displacement for the 125 lines of groups *c5* and *d* is -0.0063 Å. As the displacements for these lines show an increase for lines in the red, the mean based upon the lines in the region covered by the lines to be

tested is taken as the standard or normal displacement. The mean displacement for the 34 lines of group *e* is $+0.0142$ Å. Lines of these groups were included by Albrecht in reaching the conclusion that mutual influence displaces the violet components 0.007 Å less and the red components 0.005 Å more than the average. It is evident that a comparison, to have definite meaning, must be between lines of the same or similar groups. For lines of groups *c5* and *d* the displacements of the violet and red components of close pairs should then be of the order of -0.012 Å and 0 , respectively; for lines of group *e* $+0.007$ and $+0.019$ Å. An inspection of Tables III, IV, and V shows no indication of such systematic differences, but rather a remarkable agreement between lines in the open and lines with close companions, an agreement, however, in large measure fortuitous for arc lines of this character.

TABLE III

RELATIVE DISPLACEMENTS OF THE SOLAR AND ARC LINES OF IRON, GROUPS *c5* AND *d*
COMPANIONS TOWARD THE VIOLET IN SOLAR SPECTRUM

SECTION A. MEASUREMENTS OF WEIGHT

Å Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht	Å Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht
4024.881...	3:4	.155	-0.007	4083.433...	2:3	.439	-0.001
4083.917...	4:1	.134	-0.003	4085.730...	3:3	.298	-0.006
4154.976...	4:4	.300	-0.002	5006.306...	4:5	.410	-0.005
4187.204...	2:6	.420	-0.004	5137.558...	3:3	.308	+.002
4227.006...	20:4	.702	-0.003	-0.004	5208.776...	5:2	.180	-0.005	-0.006
4233.772...	4:6	.444	-0.004	+.002	5615.877...	2:6	.357	.000	-0.004
4247.591...	1:4	.127	-0.007	5641.667...	1:2	.401	-0.016
4250.287...	2:8	.490	-0.003	5655.715...	1:2	.320	-0.008
4469.545...	1:4	.229	.000	5659.052...	1:4	.299	-0.008	-0.003
4607.831...	1:4	.321	-0.003	Mean, 21 lines.....		.333	-0.0052
4872.332...	1:4	.220	-0.003	-.006	Standard displacement.....		-0.0050
4885.020...	2:3	.356	-0.003	+.003					

SECTION B. MEASUREMENTS WITHOUT WEIGHT

4668.331...	2:4	.088	-0.011	-0.010	5139.644*	4:4	.217	+0.003	+0.002
4957.785*	4:4	.305	+0.018	-0.004	Mean, 3 lines.....		+0.003

* Blend.

A comprehensive view of the results of the discussion based upon the behavior of 372 iron lines for which the measurements of the relative sun-arc displacements are of sufficient weight to furnish conclusions of value, and of which 142 have close companions, may

be had from Table VI, where the deviations from the mean are accounted favorable to the anomalous-dispersion hypothesis when

TABLE IV

RELATIVE DISPLACEMENTS OF THE SOLAR AND ARC LINES OF IRON, GROUPS *c5* AND *d*
COMPANIONS TOWARD THE RED IN THE SOLAR SPECTRUM

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht
4101.421...	3:2	.419	+0.002	5125.300...	1:3	.123	-0.011	-0.002
4100.215...	1:3	.394	-0.003	5139.437...	4:4	.217	-0.003	+0.006
4158.950...	5:5	.394	-0.007	5273.339...	2:3	.210	-0.004	+0.004
4101.595...	3:6	.248	-0.003	5283.862...	1:6	.470	-0.002
4100.372...	2:4	.327	-0.006	5470.778...	5:3	.345	-0.004	+0.015
4225.619...	1:3	.255	-0.005	5573.075...	1:6	.253	-0.003
4236.112...	1:8	.242	-0.003	5662.744...	1:4	.411	-0.009
4859.928...	30:4	.590	-0.004	5709.601...	5:5	.174	-0.007
4910.198...	2:3	.307	-0.005	5952.943...	1:4	.443	-0.016
4938.907...	2:4	.419	-0.005	+0.005	6246.535...	2:8	.230	-0.008
4957.480...	8:5	.305	-0.008	+0.003	6400.217...	2:8	.321	-0.007	+0.001
4982.682...	2:4	.312	-0.023	+0.013	Mean, 25 lines.....				.326 -0.0062
4985.432...	3:3	.298	-0.007	+0.009	Standard displacement.....				-0.0063
5005.896...	5:4	.410	-0.005	+0.002					

SECTION B. MEASUREMENTS WITHOUT WEIGHT

4125.776...	1:3	.074	0.000	4707.457...	2:5	.215	-0.009	+0.006*
4210.494...	3:4	.067	+0.002	+0.027	4727.582...	2:3	.094	+0.014	+0.003
Mean, 4 lines.....									

* Blend.

TABLE V

RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF IRON, GROUP *e*

COMPANIONS TOWARD VIOLET					COMPANIONS TOWARD RED								
λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun minus Arc	Albrecht				
5195.647...	4:2	.534	+0.007	-0.001	5365.669...	3:5	.527	+0.018	0.000				
5370.166...	1:6	.384	+0.013	5411.124...	1:4	.304	+0.016				
5404.357...	2:5	.320	+0.015	-0.012	5463.174...	3:3	.320	+0.014	+0.009				
5403.494...	3:3	.320	+0.018	-0.002	5598.524...	4:4	.187	+0.016	+0.008				
5330.400...	2:6	.508	+0.009	6078.710...	2:5	.517	+0.014	-0.002				
6020.401...	2:4	.169	+0.004	-0.008	Mean.....								
Mean.....		.374	+0.010	Mean.....				.371 +0.0156				
Standard displacement.....													
+0.0142 λ													

they show that the displacements of the red and violet components of close solar pairs are respectively greater and less than the mean for all lines of the group, as should be the case if mutual influence

were superposing upon the general displacement a measurable shift of the red and violet components of close pairs to the red and violet, respectively. In sign the mean result is unfavorable to the anomalous-dispersion hypothesis, but in magnitude it is practically zero, being 0.0003 A . On the assumption that the Albrecht deviations are real shifts, the displacements of lines with violet companions should exceed those of lines with red companions by $0.010-0.012 \text{ A}$. They are, however, within the limits of error, identical.

TABLE VI
RÉSUMÉ OF BEARING OF SUN-ARC DISPLACEMENTS ON MUTUAL INFLUENCE

NO. OF LINES	GROUP	COMPANION	SUN minus ARC	NORMAL	SUMMATION OF DEVIATIONS	
					Favorable	Unfavorable
56.....	<i>a, b, c4</i>	To Violet	+0.0036	+0.0038	-0.0112
29.....	<i>a, b, c4</i>	Red	+ .0038	+ .0038	0.0000
21.....	<i>c5, d</i>	Violet	- .0052	- .0050	- .0042
25.....	<i>c5, d</i>	Red	- .0062	- .0063	- .0025
6.....	<i>e</i>	Violet	+ .0110	+ .0142	- .0192
5.....	<i>e</i>	Red	+0.0156	+0.0142	-0.0070
142						
Mean deviation unfavorable to mutual influence.....						-0.0003A

The data for the lines in the second sections of Tables I, II, III, and IV are given in order to make the presentation of the Mount Wilson observations complete. Because of the recognized sources of possible error, they have not been considered in the discussion; their inclusion, however, would tend to increase the foregoing unfavorable deviation from the mean. Of these rejected lines the 17 used by Albrecht are more fully considered in the fourth division of this contribution.

The evidence of equality between the displacements of lines with companions to the violet and red, respectively, given by these 142 lines for which the conditions are favorable to the evoking of mutual influence, is clear and positive. It seems, therefore, a justifiable conclusion from this equality that mutual repulsion does not occur to a measurable amount, and that some other explanation applies to the Albrecht deviations from the mean.

III. RELATIVE SEPARATION IN SOLAR AND ARC SPECTRA

Mutual influence appearing as a quasi-repulsion requires that the difference in wave-length between a line and a closely adjacent companion be greater in the solar spectrum than when determined in terrestrial sources. If the interpretation adopted by Albrecht for his observations be the correct one, the separation in the solar spectrum, when the mean distance between the components is 0.22 \AA , should exceed that in terrestrial sources by 0.012 \AA , a quantity easily within the range of measurement. The measurements considered in this section were made upon separate plates for the sun and arc. The solar measurements, being more difficult than the corresponding ones for the arc, have been made upon plates of higher dispersion, with a scale sufficient to yield reliable results for the particular combination of intensity and spacing.¹ The conditions for the sun-arc measurements and for the determinations of separation considered in this section are so different, that it seems hardly probable that personal equation enters in the same manner or to the same degree in both, or that different observers would obtain concordant results, as they do, if personal equation introduced appreciable systematic errors. Four measurers have worked upon the sun-arc plates, and two of these have made the major part of the determinations recorded in Table VII, which form the subject-matter of this section.

For the 45 pairs the mean difference between the separations in the solar and arc spectra is zero within the limit of precision. According to the theory of mutual influence, the repulsion should be the more marked, the closer the adjacent lines. Of the pairs in Table VII, 13 have separations under 0.2 \AA , the mean being 0.148 \AA . For these pairs the mean separation in the solar spectrum is less by 0.0015 \AA than in terrestrial sources; for the remaining 32 pairs with a mean separation of 0.349 \AA , it is greater by 0.0003 \AA . In both cases the mean separations in sun and arc are practically identical.

The mean difference in separation without regard to sign, 0.003 \AA , and individual differences are larger, however, than one

¹ St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

would expect from accidental errors of measurement. The following considerations show that these large differences are systematic in sign and probably real. Three things appear distinctly in the Mount Wilson sun-arc observations: (1) in groups *a* and *b* strong lines are displaced more than weak; (2) lines of groups

TABLE VII
COMPARATIVE SEPARATION IN SOLAR AND ARC SPECTRA

λ ROWLAND	INTENSITY, ELEMENT, OR GROUP	SEPARATION				λ ROWLAND	INTENSITY, ELEMENT, OR GROUP	SEPARATION				SUN minus ARC
		V	R	Sun	Arc			V	R	Sun	Arc	
3647.561	4	12	.422	.415	+.007	4489.911	Fe	Mn	.340	.340	0.000	
3707.059	5	5	.105	.102	+.003	4654.672	<i>a</i>	<i>d</i>	.120	.132	-.012	
3711.304	4	3	.184	.184	0.000	4938.997	4	2	.423	.426	-.003	
3722.071	3	2	.102	.100	+.002	4939.416	<i>d</i>	<i>a</i>	.440	.442	+.007	
3735.014	40	4	.450	.460	-	4985.432	3	3	.296	.294	+.002	
3743.508	Fe	Ti	.108	.106	+.002	5005.890	4	5	.406	.405	+.001	
3745.717	8	6	.337	.340	-.003	5079.158	<i>e</i>	<i>b</i>	.248	.245	+.003	
3748.408	10	1	.227	.228	-.001	5107.010	4	4	.191	.193	-.002	
4000.403	2	2	.204	.200	+.004	5130.427	4	4	.211	.212	-.001	
4058.015	Fe	Mn	.170	.173	-.003	5148.222	<i>e</i>	<i>d</i>	.185	.190	-.005	
4063.430	4	20	.314	.310	+.004	5160.069	3	4	.129	.129	0.000	
4067.130	5	3	.291	.296	-.005	5208.590	Cr	Fe	.160	.170	-.001	
4079.090	3	3	.370	.370	0.000	5227.043	3	5	.322	.323	-.001	
4106.420	2	2	.171	.168	+.003	5273.330	3	2	.215	.214	+.002	
4130.078	4	6	.475	.474	+.001	5305.060	<i>e</i>	<i>a</i>	.527	.533	-.006	
4143.572	4	15	.452	.450	+.002	5404.028	<i>a</i>	<i>e</i>	.319	.303	+.016	
4154.067	4	4	.308	.310	-.002	5446.797	Ti	Fe	.332	.334	-.002	
4191.505	6	3	.240	.240	0.000	5493.174	3	3	.310	.316	+.003	
4226.904	Ca	4	.701	.710	-	5455.071	<i>e</i>	<i>a</i>	.156	.162	-.006	
4315.138	Ti	Fe	.115	.115	0.000	5470.500	<i>d</i>	<i>b</i>	.280	.284	-.004	
4427.266	Ti	Fe	.213	.215	-.002	6136.829	8	3	.373	.373	0.000	
4430.350	1	3	.425	.422	+.003	6400.217	8	2	.310	.311	-.0001	
4454.552	Fe	Ca	.399	.399	0.000							

45 lines, mean separation 0.291 Å
17 lines, sum of positive differences +.004
20 lines, sum of negative differences -.070
8 lines, zero difference000
Mean $\Delta\lambda$ Sun - $\Delta\lambda$ Arc -0.0002 Å

c and *d* are displaced to the violet; (3) lines of group *e* show the maximum displacement to the red. If, therefore, the components of a pair differ greatly in intensity or belong to different groups, whether the separation in the solar spectrum is greater or less than in the arc depends upon the configuration of the pair, provided the behavior of lines with closely adjacent companions is similar to that of free-standing lines. For example, the pair at λ 4939, with components *d* and *a*, should be wider

¹ Mt. Wilson Contr., No. 93, p. 35; Astrophysical Journal, 41, 63, 1915.

apart in the sun, while the pair at λ 4654, with components *a* and *d*, should be closer in the sun than in the arc, as the observations show. Taking as a lower limit a 50 per cent difference in intensity and considering the characteristic behavior of the groups, one can predict the relative separation in solar and arc spectra for 23 pairs as shown in Table VIII. The homogeneity of the two classes seems conclusive evidence that lines with close companions conform to the behavior of isolated lines of the same group and that their relative position in the solar spectrum is, therefore, not determined by mutual influence.

TABLE VIII
CLASSIFICATION OF SOLAR SEPARATIONS BASED UPON THE NORMAL BEHAVIOR OF
ISOLATED LINES

PREDICTED SEPARATION, SOLAR > ARC				PREDICTED SEPARATION, SOLAR < ARC					
λ Rowland	Sep.	Intensity or Group		Sep. Sun minus Sep. Arc	λ Rowland	Sep.	Intensity or Group		Sep. Sun minus Sep. Arc
		V	R				V	R	
3647.561	.422	4	12	+0.007	3735.014	.450	40	4	-0.001
4003.436	.314	4	20	+.004	3745.717	.337	8	6	-.003
4136.678	.475	4	6	+.001	3748.408	.227	10	1	-.001
4143.573	.452	4	15	+.002	4007.130	.291	5	3	-.005
4430.356	.425	1	3	+.003	4191.595	.240	6	3	-.000
4939.416	.440	d	a	+.007	4226.904	.701	20	4	-.009
5237.043	.323	3	5	-.001	4654.672	.120	8	2	-.012
5404.028	.370	6	6	+0.016	4938.997	.423	4	2	-.003
Mean	.397			+0.005 A	5070.158	.248	e	b	+.003
					5148.222	.185	e	d	-.005
					5305.060	.527	e	a	-.006
					5455.671	.150	e	a	-.006
					5476.500	.280	e	d	-.004
					6136.829	.373	8	3	-.000
					6400.217	.310	8	2	-0.001
					Mean	.325			-0.0035 A

IV. THE SYSTEMATIC DEVIATIONS OF ALBRECHT

Dr. Albrecht plotted the differences between the Rowland and the "corrected" International wave-lengths of the iron lines against wave-lengths as abscissae,¹ and found that the values for lines with violet and red companions were, respectively, above and below the curve, requiring, when the average distance between the lines and their companions was 0.22 Å, corrections of -0.005 and +0.007 Å to reduce them to the mean. He saw no reason

¹ *Astrophysical Journal*, 44, 14, 1916.

for considering these systematic deviations to be due to the rôle played by personal equation in the measurement of close pairs. He regarded them, therefore, as evidence of anomalous dispersion in the sun.

As the extensive Mount Wilson data for sun-arc displacements and for the comparative separations of the components of close pairs in solar and terrestrial spectra indicate, within the limits of precision, a total absence of mutual influence, it seems necessary to conclude either that the Mount Wilson data, though depending upon concordant results from several observers and methods, are affected by a personal equation introducing systematic errors just balancing the effects of mutual influence, or that the Rowland wave-lengths for lines in close pairs, depending upon a single observer, are systematically in error. A slight over-spacing of such pairs, displacing the components in opposite directions, would introduce an effect of the sign required by the theory. An investigation of the accuracy obtainable in the measurement of close solar pairs and of possible errors in the Rowland table has lately been carried out at this Observatory.¹ For the 30 pairs composed of lines of intensities 3 and 4 which formed the basis of the investigation, the Rowland separations are larger than those found at Mount Wilson; the sign of the Rowland errors is such as would explain the Albrecht deviations as errors in the Rowland tables. For separations of 0.274, 0.145, and 0.075 Å the errors are +0.003, +0.005, and +0.013 Å, respectively, a march of magnitude with proximity of components similar to that observed by Albrecht.

The investigation showed that spectrograms of the finest definition yield the lowest values for the separation of the components of doublets near the limit of resolution, that whatever decreases the intensity of the common region relatively to that of the continuous spectrum produces a tendency on the part of the measurer toward increased separation.

Professor Aitken finds in measures upon close double stars with 12- and 36-inch telescopes the same tendency toward over-

¹ St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

separation with increased overlap of the stellar images, as appears in measures of incompletely resolved solar pairs. I am under obligation to him for the following data:

SEPARATIONS OF DOUBLE STARS

Under 1° 12-Inch minus 36-Inch		Between 1° and 2° 12-Inch minus 36-Inch	
33 plus residuals	10 plus residuals
7 minus "	18 minus "
2 zero "	4 zero "
—	—	—	—
42 "	32 "
Mean "	Mean "
	+2.47	+0.85	
	-0.45	-1.46	
	0.00	0.00	
	+2.02	-0.61	
	+0.05	-0.02	

Concerning these data, Professor Aitken says:

The theoretical resolving power of the 12-inch is 0.39 , hence, taking distorted images into account, due to bad seeing, pairs with angular separations less than 1.0 would have little space between the two images and the closer pairs would usually be in contact, or would overlap. For pairs over 1.0 and certainly for pairs over 1.25 , there would always be clear space between the images on any night I would use for work. I think the tabulation shows that there is a systematic tendency to overmeasure small distances with the 12-inch telescope—assuming that the 36-inch measures are exact—a tendency which disappears with distances of $1''$ or more. The small negative residual (12-inch—36-inch) for the larger distances is obviously of a more accidental character than the positive residual for the smaller distances, since we have 18 minus and 10 plus signs in the one case against 33 plus and 7 minus signs in the other. This result is in harmony with many other similar comparisons I have made both of my own measures and of measures by different observers at about the same epoch with telescopes of different apertures.

These results point to an explanation of the systematic deviations observed by Albrecht, based upon errors in the Rowland wave-lengths for lines in close pairs. An extensive investigation of all the cases occurring in Albrecht's tables was undertaken. From experience gained in obtaining the data in *Contribution No. 120*, it has been possible to take advantage of the best available conditions for observations upon such difficult objects as close solar pairs. The wave-lengths of the solar lines used by Albrecht have been referred to those of the neighboring free-standing lines. All measurements have been made by two observers upon spectrograms of a scale and dispersion that experience has shown to be the most

reliable in each particular case. In every instance the data depend upon three or more concordant plates.

From the characteristic behavior of the Fe lines in the arc, it is evident that conclusions drawn from the differences between the Rowland and International wave-lengths must carry the maximum weight when based upon the stable lines of groups *a*, *b*, and *c4*. In the discussion of the Albrecht deviations, these lines, therefore, are considered separately from the unstable lines of groups *c5*, *d*, and *e*, for which the errors in the International wave-lengths are known to be systematic in sign and relatively large. According to the theory the effect of mutual influence increases with decreasing separation of the reacting solar lines; at the same time, however, the difficulties of measurement become increasingly great. As the evidence in such cases is relatively of great importance, pairs for which the separation from the companion is less than 0.1 Å are given separate consideration. The discussion of the two series of data follows under heads, A, B, C, and D.

A. Groups a, b, and c4.—The general result from Table IX is that for 88 lines the sum of the Albrecht deviations is 0.377 Å, and that of the counterbalancing Rowland errors is 0.351 Å, a mean unbalanced deviation of 0.0003 Å. The data, however, vary in reliability. For the 54 lines of greater weight, groups *a*, *b*, and *c4* in Part I, the sum of Albrecht deviations is 0.226 Å, that of the Rowland errors, 0.255 Å. That the Albrecht deviations and the Rowland errors are mutually explanatory appears clearly from the parallelism between them. In Table X the largest Albrecht deviations are in the first column and the smallest in the third, the Rowland errors for the same lines being in the second and fourth columns of the two parts of the table. For the 27 lines showing the smallest Albrecht deviations the mean mutual influence is zero, and the Rowland error correspondingly small. If the larger deviations are due to mutual influence, a lack of parallelism should be conspicuous for them, but here the correspondence is particularly close. Among the 27 lines with the largest deviations, discrepancies are pronounced for λ 5195 and λ 5476 indicated by "?" in the table. These lines have companions to the red for which, on the hypothesis of mutual influence, large displacements are characteristic. Their

TABLE IX

THE ALBRECHT SYSTEMATIC DEVIATIONS AND ERRORS IN THE ROWLAND TABLE
PART I. LINES OF GROUPS a , b , AND $c4$. SECTION A. COMPANION TO VIOLET

λ Rowland	Int. Ratio Comp. to Line	$\Delta\lambda R.$	λ Mt.W.	Mt.W.-R.	Albrecht	Remarks
3647.988	4:12	0.427	.986	-0.002	+0.002	
3680.060	2: 9	.248	.062	- .007	- .007	
3737.281	5:30	.222	.280	- .001	+ .003	
3746.058	8: 6	.341	.054	- .004	- .008	
3748.650	10: 1	.242	.646	- .004	- .008	
3887.196	3: 7	.254	.198	+ .002	- .000	
3888.671	2: 5	.111	.663	- .008	- .008	
3895.803	3: 7	.220	.805	+ .002	- .000	
3900.413	6:10	.527	.410	- .003	- .004	
4132.235	2:10	.135	.222	- .013	- .012	
4134.840	3: 5	.251	.836	- .004	+ .006	
4144.038	4:15	.406	.029	- .000	- .005	
4147.836	2: 4	.334	.831	- .005	+ .002	
4191.843	6: 3	.248	.830	- .004	+ .003	
4291.630	2: 2	.354	.625	- .005	+ .007	
4308.081	3: 6	.174	.065	- .016	- .006	
4315.262	3: 4	.124	.252	- .010	- .007	
4427.482	2: 5	.216	.476	- .006	.000	
4531.327	2: 5	.204	.326	- .001	- .002	
4556.306	3: 4	.243	.309	+ .003	+ .002	
4592.840	2: 4	.133	.831	- .009	- .010	
4787.003	3: 2	.270	.904	- .009	- .012	
4789.840	2: 3	.321	.840	- .009	- .012	
4930.868	2: 3	.452	.867	- .001	- .002	
5028.308	1: 2	.369	.310	+ .002	.000	
5041.255	3: 4	.186	.250	- .005	- .002	
5041.936	2: 4	.141	.943	+ .007	+ .001	
5079.400	3: 4	.251	.402	- .007	- .006	
5098.885	1: 3	.134	.876	- .009	- .005	
5107.823	4: 4	.204	.820	- .003	- .004	
5167.678	15: 5	.181	.669	- .009	- .016	
5227.302	3: 5	.319	.355	- .007	- .003	
5270.558	3: 4	.130	.541	- .017	- .020	
5273.558	3: 2	.319	.555	- .003	- .014	
5333.080	1: 4	.240	.088	- .001	+ .005	
5305.590	5: 3	.527	.593	- .003	+ .003	
5447.130	2: 6	.333	.125	- .005	+ .003	
5455.834	2: 4	.163	.810	- .024	- .007	
5191.779	6: 9	0.386	.775	-0.004	-0.004	
Mean, 39 lines				-0.0054	-0.0039	

SECTION B. COMPANION TO RED

3705.708	2: 9	0.141	.714	+0.006	+0.007	
3735.014	4:40	.471	.013	- .001	+ .004	
3743.508	2: 6	.118	.511	+ .007	+ .006	
3834.364	4:10	.142	.377	+ .013	+ .009	
4337.210	3: 5	.509	.216	.000	+ .002	
4454.552	5: 3	.401	.553	+ .001	+ .003	
4461.818	3: 4	.347	.824	+ .006	+ .008	
4480.911	3: 4	.342	.910	- .001	+ .004	
4679.027	2: 6	.382	.029	+ .002	+ .005	
5107.619	4: 4	.204	.627	+ .008	+ .008	
5195.113	2: 4	.534	.113	.000	+ .006	Quality excellent in sun and arc*
5208.596	2: 5	.180	.598	+ .002	+ .003	Arc λ depends on unstable standards
5328.236	2: 8	.279	.235	- .001	- .007	
5476.500	3: 1	.278	.496	- .004	+ .011	Weak line 0.11 to violet vitiates measures
6136.829	3: 8	0.381	.835	+0.006	+0.004	
Mean, 15 lines				+0.0029	+0.0049	

* *Astrophysical Journal*, 44, 21 and 30, 1916.

TABLE IX—Continued

PART II. LINES OF GROUPS *c5* AND *d*. SECTION A. COMPANION TO VIOLET

λ Rowland	Int. Ratio Comp. to Line	$\Delta\lambda$	λ Mt.W.	Mt.W.—R.	Albrecht	Remarks
4227.606....	20:4	0.702	.506	—0.010	—0.004	
4233.772....	4:6	.444	.769	—.003	+.002	
4873.332....	1:4	.230	.329	—.003	—.006	
4957.785....	5:8	.305	.785	.000	—.004	
4985.730....	3:3	.298	.729	—.001	+.003	
5000.306....	4:5	.410	.303	—.003	—.002	
5130.644....	4:4	.217	.642	—.002	+.002	
5208.776....	5:2	.180	.766	—.010	—.006	
5603.186....	3:4	.103	.178	—.008	.011	
5615.877....	2:6	.357	.870	—.007	—.004	
5050.052....	1:4	0.399	.042	—0.010	—0.003	
Mean, 11 lines.....				—0.0052	—0.0030	
SECTION B. COMPANION TO RED						
4191.595....	3:6	0.248	.598	+.003	+.011	
4637.685....	4:5	.508	.685	.000	+.004	
4707.457....	2:5	.215	.459	+.002	+.006	
4938.997....	2:4	.419	.996	—.001	+.005	
4957.480....	8:5	.305	.481	+.001	+.003	
4982.682....	2:4	.312	.682	.000	+.013	
4985.432....	3:3	.298	.433	+.001	+.009	
5005.896....	5:4	.410	.898	+.002	+.002	
5125.300....	1:3	.123	.206	—.004	—.002	
5139.427....	4:4	.217	.431	+.004	+.006	
5273.339....	2:3	.219	.343	+.004	+.004	
5476.778....	5:3	.345	.775	—.003	+.015	
6400.217....	2:8	0.321	.221	+.004	+.001	
Mean, 13 lines.....				+0.0010	+0.0059	

PART III. LINES OF GROUP *c*. SECTION A. COMPANION TO VIOLET

5195.647....	4:2	0.534	.644	—0.003	—0.001	
5404.357....	2:5	.320	.344	—.013	—.012	
5403.404....	3:3	.320	.488	—.006	—.002	
5504.884....	4:1	.193	.882	—.002	+.009	
6020.401....	2:4	0.169	.301	—0.010	—0.008	
Mean, 5 lines.....				—0.0068	—0.0028	

SECTION B. COMPANION TO RED

5365.069....	3:5	0.527	.068	—0.001	0.000	
5403.174....	3:3	.320	.169	—.005	+.009	
5508.524....	4:1	.187	.525	+.001	+.008	
6008.186....	6:4	.590	.186	.000	+.012	
6078.710....	2:5	0.517	.707	—0.003	—0.002	
Mean, 5 lines.....				—0.0016	+0.0054	

significance in this case is, however, open to question, for λ 5195, intensity 4, is 0.534 Å from its companion of intensity 2, and λ 5476, intensity 1, stands between two lines, a weaker one, 0.113 Å to the violet, a stronger one, 0.278 Å to the red. The arc wavelength of λ 5195 is derived from unstable standards of group *d*,

and is subject to the errors attaching to them. Of the arc wavelength of λ 5476 St. John and Ware remark, "Near diffuse line λ 5476.587. Separation difficult." Notes upon the observing records for its solar wave-length indicate that the measurements are influenced by the weak violet companion which, even on plates of high dispersion, is contiguous to the violet edge and tends to produce a fictitious shift to the violet.

TABLE X
CORRESPONDENCE BETWEEN ALBRECHT DEVIATIONS AND ROWLAND ERRORS BASED
UPON LINES OF GROUPS *a*, *b*, AND *c4* IN TABLE IX

COMPANION TO VIOLET				COMPANION TO RED			
20 Largest Deviations		19 Smallest Deviations		7 Largest Deviations		8 Smallest Deviations	
Albrecht	AMt.W. minus AR.	Albrecht	AMt.W. minus AR.	Albrecht	AMt.W. minus AR.	Albrecht	AMt.W. minus AR.
-.007	-.007	+.002	-.002	+.007	+.006	+.004	-.001
-.008	-.004	+.003	-.001	+.006	+.007	+.002	.000
-.008	-.004	.000	+.002	+.009	+.013	+.003	+.001
-.008	-.008	.000	+.002	+.008	+.006	+.004	-.001
-.004	-.003	+.006	-.004	+.008	+.008	+.005	+.002
-.012	-.013	+.002	-.005	+.006?	-.000?	+.003	+.002
-.005	-.009	+.003	-.004	+.011?	-.004?	-.007	-.001
-.006	-.016	+.007	-.005	+.004	+.006
-.007	-.010	.000	-.006				
-.010	-.009	-.002	-.001				
-.012	-.009	+.002	+.003				
-.012	-.009	-.002	-.001				
-.006	-.007	.000	+.002				
-.005	-.009	-.002	-.005				
-.004	-.003	+.001	+.007				
-.016	-.009	+.003	-.007				
-.026	-.017	+.005	-.001				
-.014	-.003	-.003	-.003				
-.007	-.024	+.003	-.005				
-.004	-.004				
SUMS AND MEANS							
-.0.181	-.0.177	+.0.028	-.0.034	+.0.055	+.0.036	+.0.018	+.0.008
-.0.0090	-.0.0088	+.0.0015	-.0.0018	+.0.0079 (7)	+.0.0051 (7)	+.0.0022	+.0.0010
				+.0.0076 (5)	+.0.0080 (5)		

B. Groups *c5*, *d*, and *e*.—As to the 34 lines of the unstable groups *c5*, *d*, and *e*, the situation is so complicated that an attempt to follow its intricacies would be profitless. Four considerations are involved: (1) The systematic errors in the Rowland values tending to account for the deviations found by Albrecht. The average error is 0.003 Å, somewhat smaller than for lines of groups

¹ Mt. Wilson Contr., No. 61, p. 28; *Astrophysical Journal*, 36, 41, 1912.

a, *b*, and *c*₄. Such a difference is consistent with the relative appearance of the lines in arc and solar spectra, for the lines of groups *c*₅, *d*, and *e*, diffuse, unsymmetrical, and difficult to measure in the arc, are, in general, the sharpest and most accurately measurable in the solar spectrum. (2) The undetermined systematic errors in the International wave-lengths of these lines, positive for groups *c*₅, and *d*, negative for group *e*. In recognition of these errors standard conditions for the arc were adopted at the Bonn meeting of the International Solar Union and redeterminations of the iron standards have been undertaken in various laboratories. Preliminary results at this Observatory indicate that the errors are systematic and of the sign expected. (3) The systematic errors in the data for pressure-shift. At the time of publication of the pressure data, the sensitiveness of these lines to varying arc conditions had not been recognized and the needed precautions against pole-effect were not taken. Recent observations under standard arc conditions give 0.008 Å per atmosphere for groups *c*₅ and *d* instead of the 0.022 Å previously published. For the lines of group *e* the later result is +0.002 Å per atmosphere instead of the negative value -0.016 Å.¹ (4) Albrecht's effort to make the International and Rowland wave-lengths homogeneous by "correcting" the International values to a pressure of half an atmosphere. That this operation still left systematic differences appears in Table XI based upon data from Albrecht's Table I. For reasons given later and because the points fall so far from the curve that they have had little if any influence on its course, λ 4204, λ 4210, and λ 5371 have not been taken into account.

A reference-curve based upon $\Delta\lambda'$ for all lines is on the average too high for the lines of groups *c*₅ and *d*, whose proportional representation is small. This tends to increase their deviations to the violet and to decrease those to the red. Such a tendency shows in Part II of Table IX, where the mean deviation to the violet is 0.006 Å, that to the red 0.003 Å, while the Rowland errors are 0.001 and 0.005 Å, respectively. A lowering of the curve by 0.004 Å would bring the deviations and errors into practical agreement. As a systematic effect of this order is indicated by the

¹ *Mt. Wilson Contr.*, No. 106, p. 16; *Astrophysical Journal*, 42, 231, 1915.

negative residuals in Table XI, curves have been drawn for the lines of groups *c*5 and *d* alone, using $\Delta\lambda$ and $\Delta\lambda'$ from Albrecht's Table I as ordinates. The mean displacement for the same lines of these groups with companions to the red, deduced from either curve, is 0.002 Å instead of 0.006 Å, a result in close agreement with the corresponding Rowland error of 0.001, while the mean for the lines with companions to the violet remains practically unchanged. This systematic effect, introduced by referring the lines of groups *c*5 and *d* to a curve based largely upon lines of other

TABLE XI
SYSTEMATIC VARIATION FROM HOMOGENEITY IN $\Delta\lambda'$

Region	Group	$\Delta\lambda$	$\Delta\lambda'$	$\Delta\lambda'$ Group <i>c</i> 5, <i>d</i> minus $\Delta\lambda'$ Group <i>a</i> , <i>b</i>
4200-4300	{ <i>c</i> 5, <i>d</i>159	.165	+0.001
	{ <i>a</i> , <i>b</i>163	.164	
5000-5100	{ <i>c</i> 5, <i>d</i>165	.173	-0.007
	{ <i>a</i>178	.180	
5100-5200	{ <i>d</i>155	.168	-0.004
	{ <i>a</i>170	.172	
5200-5300	{ <i>d</i>164	.175	-0.002
	{ <i>a</i>175	.177	
5300-5400	{ <i>d</i>177	.187	-0.009
	{ <i>a</i>194	.196	
6300-6500	{ <i>d</i>195	.212	-0.004
	{ <i>b</i>212	.216	

groups, is apparently of a sign and magnitude to account for the inequality of the displacements for lines with red and violet companions, which Albrecht considered the principal objection to an explanation based upon personality in the Rowland measures.¹

C. *Lines less than 0.1 Å from companion.*—Of the 104 lines considered by Albrecht, the remaining 16 are separated from the companion line by less than 0.1 Å. The data relative to these very close pairs are given in Table XII. The wide range in the deviations, +0.048 to -0.002 Å for the 6 lines with companions

¹ *Astrophysical Journal*, 41, 355, 1915.

TABLE XII
LINES LESS THAN 0.1 Å FROM COMPANION
PART I. COMPANION TO VIOLET

λ Rowland	Int.	Element and Class	$\Delta\lambda R$	λ Mt.W.	Mt.W. minus Rowland	Albrecht	Remarks
3722.630 (.692)	3...	Ni	0.090				
.729	10...	Ti-Fe, a		.714	-0.015	-0.014	
	6...						
4204.204	2...	Ti	0.097				
	301	5...	Fe, b	.287	-0.014	-0.005	
4407.810	2...	V	0.061				
	871	4...	Fe, c4	.878	+0.007	+0.011	
4668.243	2...		0.088				
	331	4...	Fe, d	.322	-0.009	-0.010	
4727.582	3...	Fe	0.094				
	676	2...	Mn	.662	-0.014	-0.029	
4878.313	3...	Ca	0.094				
	407	4...	Fe, c5	.406	-0.001	-0.008	
5202.439	2...	Fe?	0.077				
	516	4...	Fe, b	.510	-0.006	-0.006	
5371.656	4...	Cr?	0.078				
	734	3...	Fe, a	.692	-0.042	-0.042	Misidentification by Rowland. Not a pair in solar spectrum but a single line due to Fe
6147.950	2...		0.090				
	8.040	3...	Fe, d	.038	-0.002	-0.005	
6254.382	1...		0.074				
	456	5...	Fe, b	.479	+0.023	+0.017	Doubtful, Albrecht

PART II. COMPANION TO RED

3997.547	4...	Fe, b	0.091	.549	+0.002	+0.001	
	638	2...					
4204.101	3...	Fe, b	0.062	.106	+0.005	+0.048	Only incipiently separated from red companion by the highest resolving power used. Blend in solar spectrum. High dispersion shows weak line contiguous to violet edge. Burns gives two lines in arc 4203.953, intensity 1, and .985, intensity 3, of which the stronger alone was used by Albrecht. Both things contribute to the abnormal displacement to violet
	163	4...	La				

TABLE XII—Continued
PART II (Continued). COMPANION TO RED

λ Rowland	Int.	Element and Class	ΔAR	λ Mt.W.	Mt.W. minus Rowland	Albrecht	Remarks
4310.494 561	4...	Fe, c5	0.067	499	+0.005	+0.027	Resolution incipient. Blend in solar spectrum. With high dispersion a line contiguous to the violet edge is conspicuous. The measurements, being upon the blend of the two, give the apparent shift to the violet.
	3...	
4476.185 253	4...	Fe, δ	0.008	182	-0.003	+0.008	Resolution incipient. In the arc, "Unreliable. Hazy on the red edge." St. John and Ware; "Ur." Goos; "Not suitable for standard," Janick; "Close double in vacuum," Babcock
	3...	Ag		
4727.582 676	3...	Fe, δ	0.094	584	+0.002	+0.003
	2...	Mn		
5012.252 335	4...	Fe, s	0.083	250	-0.002	-0.002
	1...	

to the red, and -0.041 to $+0.017$ Å for the 10 lines with companions to the violet, indicates at once the low weight to be assigned to the data as evidence of mutual influence. Three pairs of lines, companions to the red, $\lambda 4204$, $\lambda 4210$, and $\lambda 4476$, are at the limit of spectrographic resolution for lines of these intensities. The probable errors are therefore large.¹ The record of observations in the last column furnishes sufficient grounds for omitting them from a definitive discussion. The three other pairs with companions to the red are measurable with high precision and the conditions are apparently favorable to the appearance of mutual influence; the mean displacement to the violet, however, is minute, 0.0007 Å, and balanced by the Rowland errors.

Of the 10 lines with companions to the violet, 4 are of doubtful or no weight in a definitive discussion; $\lambda 3722$ itself is a blend and is in a complex very difficult of resolution; the arc wave-length of $\lambda 4727$ is uncertain; the indicated pair at $\lambda 5371$ is a single line; $\lambda 6254$ is not separable from its companion upon a fourth-order spectrogram, upon which the head of the oxygen band at $\lambda 6276$ is completely resolved, a resolution not reached by the Rowland

¹ St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

plates for that region. The deviations for the remaining 6 lines are balanced by Rowland errors.

The most favorable condition for the appearance of mutual influence is in the case of lines with companions at the closest distance consistent with precision of measurement, as the probability is then greatest that the affected lines are situated upon a steep portion of the "anomaly-curve" due to the adjacent line.¹ Observations upon lines that, aside from closeness to a companion line, are free from known sources of error are consequently of crucial importance in their bearing upon the hypothesis of anomalous

TABLE XIII
CORRESPONDENCE BETWEEN ALBRECHT DEVIATIONS AND ROWLAND ERRORS FOR THE
SMALLEST MEASURABLE SEPARATIONS, LINES FREE
FROM EXTRANEOUS SOURCES OF ERROR

COMPANION TO RED				COMPANION TO VIOLET			
λ Rowland	$\Delta\lambda$	$\lambda\text{Mt.W.}$ minus $\lambda\text{R.}$	Albrecht	λ Rowland	$\Delta\lambda$	$\lambda\text{Mt.W.}$ minus $\lambda\text{R.}$	Albrecht
3907.547...	0.085	+0.002	+0.001	4204.301...	0.078	-0.014	-0.005
4737.582...	0.084	+0.002	+0.003	4407.871...	.068	+ .007	+ .011
5012.252...	0.082	-0.002	-0.002	4608.331...	.077	- .009	- .010
Means.....	0.083	+0.0007	+0.0007 A	4878.407...	.092	- .001	- .008
				5202.516...	.077	- .006	- .006
				6148.040...	0.099	-0.002	-0.005
				Means....	0.082	-0.0042	-0.0038 A

dispersion in the solar atmosphere, and failure here would indicate that, within the present attainable precision in measurement, anomalous dispersion does not systematically alter the relative positions of the Fraunhofer lines. Data for the 9 lines fulfilling these conditions are given in Table XIII. Their average separation is 0.082 Å; ratio of companion to line, 3.2:2.7; weight, according to Albrecht, 1.7. The displacement to the violet for the lines with companions to the red should be larger than the displacement to the red for lines with companions to the violet. The deviation to the violet is, however, vanishingly small, much less than that to the red, and both are balanced by the corresponding Rowland errors.

¹ *Astrophysical Journal*, 43, 53, note, 1916.

D. *Correlation.*—The correspondence between the Albrecht deviations and the Rowland errors appears distinctly in Tables X and XIII. A line-to-line comparison for these 63 lines, to which a maximum of weight may be ascribed, shows a striking parallelism. A direct correspondence between the 104 Albrecht deviations and the Rowland errors for the same lines is shown to be a practical certainty by the large correlation coefficient $+0.56$ and its small probable error ± 0.05 .

The facts observed by Albrecht and the corresponding results for this investigation are strikingly complementary.

ALBRECHT

Fraunhofer lines as given in Rowland's table are displaced when they have close companions.

a) The displacement is to the violet when the adjacent line is to the red.

b) The displacement is to the red when the adjacent line is to the violet.

c) The displacement in (a) is greater than in (b).

d) The displacement increases as the separation between the lines diminishes.

e) The displacement is inappreciable for separation somewhat under 0.7 \AA .

ST. JOHN

Rowland wave-lengths of lines with close companions are systematically in error.

a') The sign of the error is negative when the adjacent line is to the red.

b') The sign of the error is positive when the adjacent line is to the violet.

c') A systematic excess in the Albrecht displacement to the violet for lines of groups *c5* and *d* tends to make (a') greater than (b').

d') The Rowland error increases as the separation of the lines diminishes.

e') The Rowland errors cease to be systematic for separation somewhat under 0.5 \AA .

There seems to be no explanation of the *pari passu* march of the Albrecht observations and those recorded in Section IV of this paper other than that they are two phases of the same phenomenon.

V. ADJACENT LINES DUE TO DIFFERENT ELEMENTS

Under the title, "Mutual Repulsion of Contiguous Spectrum Lines," Sir Joseph Larmor says:

Thus *very close spectrum lines ought to repel each other*, to a degree that experience alone can reveal.

But this conclusion implies that the adjacent lines represent independent vibrations. If they are two components of a single compound vibration of the molecule, the argument is not applicable.

If an iron line in the solar spectrum has a very close adjacent line *due to another substance*, while in the arc spectrum that substance is not present, then a displacement of the solar line relative to the arc line may be looked for.¹

If such a mutual repulsion is operative to a measurable degree in the solar atmosphere, the sun-arc displacements of the Fe lines should be greater for the red and less for the violet components of a pair when the adjacent line is due wholly or in part to another substance than when the influencing line is iron or weak and unknown.² The most dependable results should be given by lines of groups *a*, *b*, and *c*₄, both because of their stability in the arc and because of their greater number. For these 85 lines a residual of 0.002 Å would have considerable weight. For the lines of groups *c*₅, *d*, and *e*, the errors due to their sensitiveness to arc conditions are relatively large, and definite indications of characteristic behavior require correspondingly larger residuals for a like number of lines. The comparison between the displacements of 72 Fe lines, Table XIV, under the influence of adjacent lines in which other substances are concerned with the 70 for which mutual influence is supposed to be smaller shows no differences greater than the limit of error.

Among the 45 pairs in Table VII for which the separations were measured in solar and arc spectra are 8 close pairs of lines that fulfil

¹ *Observatory*, 497, 103, 1916.

² NOTE.—According to Sir Joseph Larmor an increase of refractive power is accompanied by a lowering of the aethereal elasticity which results in an increase of the free period and consequently a real displacement of the red component of a close solar pair toward longer wave-length. Similar considerations indicate a displacement of the violet component toward shorter wave-length. Such changes in vibration-frequency occur only when the anomalous refractive power is due to an adjacent line originating in an independent vibrating system, that is, in general, to a line of another element. The view of Professor Julius is that, owing to a mutual influence of two close Fraunhofer lines, the refractive power of the medium for the spectral region between them is decreased, less R- and V-light is anomalously refracted or scattered away from the observer on their opposed edges, and the centers of gravity of the dispersion bands are consequently farther apart. As the mutual influence consists simply in the superposition of the anomalous refractive powers due to the two absorption lines, the displacements should be independent of the elements involved. Julius adds the proviso that the elements coexist in the mixture at the same levels.

the Larmor conditions, as the arc wave-lengths for the different elements were determined in separate arcs. For these 8 pairs, mean separation 0.23 Å (Table XV), there is no systematic difference between the separations in the sun and in independent

TABLE XIV
SUN-ARC DISPLACEMENTS IN RELATION TO ORIGIN OF THE ADJACENT LINE

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Companion	Group	No. of Lines	Sun-Arc	No. of Lines	Sun-Arc	Remarks
To violet.....	a, b, c_4	30	$+0.0040$	26	$+0.0031$	Favorable by 0.001 Å
To red.....	a, b, c_4	19	$+0.0048$	10	$+0.0031$	Unfavorable by 0.001 Å
To violet.....	c_5, d	10	-0.0031	11	-0.0072	Three abnormal values included
		9	-0.0037	9	-0.0047	Three abnormal values omitted
To red.....	c_5, d	10	-0.0066	15	-0.0000	Equal within limit of error
To violet.....	e	1	$+0.013$	5	$+0.011$	Equal within limit of error
To red.....	e	2	$+0.016$	3	$+0.015$	Equal within limit of error

(4) Adjacent line wholly or in part not iron.

(6) Adjacent line iron or weak and unknown.

TABLE XV
SEPARATIONS IN THE SOLAR SPECTRUM COMPARED WITH SEPARATIONS DETERMINED FROM INDEPENDENT ARCS WHEN THE ADJACENT LINE IS DUE TO ANOTHER ELEMENT

λ ROWLAND	ELEMENTS	SEPARATION		SEP. SUN MINUS SEP. ARC
		Sun	Arc	
3743.508.....	Fe, Ti	.108	.106	$+0.002$
4058.915.....	Fe, Mn	.170	.173	-0.003
4315.138.....	Ti, Fe	.115	.115	.000
4427.266.....	Ti, Fe	.213	.215	-0.002
4454.552.....	Fe, Ca	.399	.399	.000
4489.911.....	Fe, Mn	.340	.340	.000
5208.596.....	Cr, Fe	.169	.170	-0.001
5446.797.....	Ti, Fe	.332	.334	-0.002
Means.....		.231	.232	-0.0008

terrestrial sources, and the absence of mutual repulsion is still shown by the 4 closer pairs, mean separation 0.14 Å. It is evident that the effect is not within the present means of observation.

In a recent number of the *Astrophysical Journal*¹ Albrecht deduces the following unweighted results from the systematic

¹*Astrophysical Journal*, 44, 1, 1916.

deviations between the Rowland and the "corrected" International wave-lengths for Fe lines with close companions:

COMPANION NOT FE	Sum of Deviations	COMPANION FE	Sum of Deviations
20 lines, comp. to R. 0.008	0.160	18 lines, comp. to R. 0.005	0.090
41 " " " V. 0.0047	0.193	23 " " " V. 0.003	0.060
61 " total sum	0.353	41 " total sum	0.168
Mean	0.0055	Mean	0.0041

and concludes that "this material reduction of the displacement for pairs in which both components are due to iron is distinctly in line with Larmor's theory."

The data show an apparent Larmor effect of 0.0014 Å. There are included, however, 3 lines whose evidence in a definitive discussion is open to question, namely, $\lambda 4204$, $\lambda 4210$, and $\lambda 5371$. The Albrecht deviations for them are 0.048, 0.027, and 0.042 Å. These average 8 times the mean for the 102 lines. Aside from their extraordinary magnitudes, other grounds for assigning them a very low weight have been given in a previous section and are here briefly restated. $\lambda 5371$ is not in a solar pair with Cr but is a single line due to iron; $\lambda 4204$ and $\lambda 4210$ are only partially resolved from their red companions even on excellent fifth-order spectrograms; each has, moreover, a hitherto unnoted weak line contiguous to its violet edge; $\lambda 4204$ is double in the arc, the red component only was used in the comparison, and $\lambda 4210$ is subject to pole-effect. These conditions all combine to produce abnormal violet displacements. If these three lines are omitted, the indication of a Larmor effect disappears, as the following tabulation shows:

COMPANION NOT FE	Sum of Deviations	COMPANION FE	Sum of Deviations
20 - 2 = 18 lines comp. to R.	0.085	18 lines comp. to R.	0.099
41 - 1 = 40 " " " V.	0.151	23 " " " V.	0.069
58 " total sum	0.236	41 " total sum	0.168
Mean	0.0041	Mean	0.0041

Other lines giving extraordinary values, namely, $\lambda 3722$, $\lambda 5167$, $\lambda 5270$, and $\lambda 6254$, are practically in the same category as the three mentioned above. Their omission does not change the evidence against the Larmor effect. The inconsistency between

0.004 Å given by the 99 lines and 0.039 Å given by 3 lines is so pronounced that one seems compelled to choose between them. The straight means only are here considered, as the system of weighting used by Albrecht unfortunately assigns the greater weight to observations presenting the greater difficulties of execution. For example, a combined weight of 8 is assigned to these 3 questionable lines in obtaining his final result, the average weight of the 102 lines being 1.25, so that in the weighted mean their influence is more than doubled.

Albrecht remarks further:

As the displacement has not entirely disappeared for pairs in which both lines are due to iron, we must conclude that the components of these pairs represent only in part actual physical connection in the molecule, and in part entirely independent vibrations.¹

It is implied that if the components are due to one compound vibration, that is, to a single element, and originate at the same level, no displacement would occur. This seems to be a relinquishment of the Julius point of view upheld in the former paper and opposed to the assumption of Julius that mutual influence is operative only when the lines originate at the same level.²

VI. PRESSURE IN THE SOLAR ATMOSPHERE

The pressure of 0.5 atmosphere found by Albrecht for the solar atmosphere was based upon the pressure-shifts for the iron lines, published previously to the appearance of his paper. *Contribution No. 106* from this Observatory,³ in which attention was called to the errors in the published data for groups *c*, *d*, and *e*, due to pole-effect, was issued later. Using the old pressure-shifts of +0.022 Å for groups *c* and *d*, and -0.016 Å for group *e*, Albrecht obtained results of which he says:

. . . . The correction of the wave-lengths in the International system to a pressure of 0.5 atmosphere has brought in toward the curve the lines which are strongly affected by pressure—namely, those of groups *c*, *d*, and *e*. Without the application of these corrections, the lines of groups *c* and *d* are decidedly

¹ *Astrophysical Journal*, 44, 8, 1916.

² *Ibid.*, 43, 49, 1916.

³ St. John and Babcock, "A Study of the Pole-Effect in the Iron Arc," *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, 42, 231, 1915.

below the curve, and those of group *e* are above the curve. This "drawing in" of the points toward the curve is gratifying, as it shows that the reduction of the wave-lengths in the International system to a pressure of 0.5 atmosphere has made them practically homogeneous with Rowland's wave-lengths in the sun—except, of course, for the systematic differences between the two systems as represented by the curve.¹

A homogeneity attained by applying corrections based upon pressure data in which the errors are comparable to the original lack of homogeneity can hardly be regarded, in the light of fuller knowledge, as a justification of the method. That the practical homogeneity is still affected by systematic deviations for the lines of groups *c*₅ and *d* is apparent in Table XI, and the lack of homogeneity, when the later pressure-shifts are used, is shown in Table XVI. Even an assumed pressure of zero fails to make them homogeneous with the Rowland wave-lengths, but they become so at a

TABLE XVI
NON-HOMOGENEITY OF $\Delta\lambda'$ AT 0.5 ATMOSPHERE

REGION	GROUP	$\Delta\lambda$	$\Delta\lambda'$ AT 0.5 ATM.		$\Delta\lambda'$ AT 5 ATM.
			Old Pressure Observations	Later Pressure Observations	
5202-5341	{ <i>a</i> <i>d</i>	0.182 0.166	0.184	0.184	0.198
			0.177	0.170	0.198
5365-5476	{ <i>a</i> <i>e</i>	0.208 0.217	0.210	0.210	0.224
			0.209	0.218	0.225

pressure of 5 atmospheres, a pressure in the solar atmosphere inconsistent, however, with the long series of sun-arc observations with which comparison is made in Table XVII. The discrepancies between the observed and calculated displacements are so large that it is evident that a homogeneity depending upon a correction for pressure only does not rest upon a substantial basis. In fact, so many elements enter into the problems raised by the differences between solar and arc wave-lengths that it seems advisable to obtain a much wider range of sun-arc and other data before entering upon a definitive discussion of the question of pressure in the solar

¹ *Astrophysical Journal*, 41, 351, 1915.

atmosphere. Of four suggested explanations of the differences between solar and arc wave-lengths, (1) pressure, (2) motion in line of sight, (3) difference in the gravitational fields of the sun and earth, (4) anomalous dispersion, Albrecht considers only the first. The omission of the fourth, a fundamental deduction from the hypothesis according to Julius, is remarkable in a paper purporting to establish anomalous dispersion in the sun.

TABLE XVII
SUN-ARC DISPLACEMENTS INCONSISTENT WITH A PRESSURE OF
5 ATMOSPHERES IN THE SUN

Group	No. of Lines	Calculated at 5 Atm.	Observed	O-C
<i>a</i> and <i>b</i>	211	+0.016	+0.004	-0.012
<i>c</i> ₅ and <i>d</i>	125	+0.032	-0.006	-0.038
<i>e</i>	34	+0.007	+0.014	+0.007

The outstanding indications that appear to be established by solar observations are that no one cause furnishes a satisfactory explanation of the differences between the wave-lengths in solar and arc spectra, and that pressure, though intimately concerned, does not play the supremely predominant rôle formerly attributed to it. A conclusion that the pressure in the solar atmosphere where the Fe lines originate is 0.5 atmosphere appears questionable when no account has been taken of variation with the solar intensity of the lines and other suggested effects.

I wish to express my appreciation of the assistance in this long and exacting investigation rendered by Miss Ware, whose unflagging interest and conscientious work have made the investigation possible and greatly increased the weight of the observations.

SUMMARY AND CONCLUSIONS

1. The mean sun-arc displacements for 211 Fe lines of the stable groups *a*, *b*, and *c*₄ is +0.0038 Å; that for 56 lines with companions to the violet, mean separation 0.275 Å, is +0.0036 Å; that for 29 lines with companions to the red, mean separation 0.320 Å, is +0.0038 Å.

2. The mean sun-arc displacement for 125 lines of groups *c*₅ and *d* is -0.0063 \AA ; that for 25 lines in the same region with companions to the red is -0.0062 \AA . The mean displacement for 21 lines with companions to the violet is -0.0052 \AA ; that for all *c*₅ and *d* lines in the same region is -0.0050 \AA .

3. The mean sun-arc displacement for 34 lines of group *e* is $+0.0142 \text{ \AA}$; that for 5 lines with companions to the red and 6 with companions to the violet is $+0.0156$ and $+0.0110 \text{ \AA}$, respectively.

4. The mean separation in the solar spectrum for 45 pairs is, within the limits of error, identical with that in arc spectra.

5. The behavior of lines with companions is like that of similar isolated lines. Whether the separation in the solar spectrum is greater or less than in arc spectra depends upon the configuration of the pair. For 8 pairs of the 45 it should be larger, for 15 smaller; it is respectively 0.005 \AA greater and 0.0035 \AA less than in arc spectra.

6. For 54 lines of groups *a*, *b*, and *c*₄, separation $>0.1 \text{ \AA}$, the sum of the systematic displacements found by Albrecht is 0.226 \AA , that of the counterbalancing errors in the Rowland wave-lengths is 0.255 \AA .

7. For 34 lines of groups *c*₅, *d*, and *e*, separation $>0.1 \text{ \AA}$, the sum of the systematic displacements is 0.151 \AA ; corrected for systematic error it is 0.099 \AA ; that of the balancing errors is 0.096 \AA .

8. Of the 16 remaining lines, separation $<0.1 \text{ \AA}$, 7 are without weight in a definitive discussion. The sum of the systematic displacements for the remaining 9 lines, separation from companions 0.082 \AA , is 0.025 \AA ; that of the balancing Rowland errors is 0.027 \AA .

9. The coefficient of correlation between the 104 displacements attributed by Albrecht to mutual influence and the 104 errors in Rowland wave-lengths found in this investigation is $+0.56 \pm 0.05$.

10. The sun-arc displacements for iron lines are independent of the origin of closely adjacent lines.

11. The separations of iron lines from those due to another substance are the same in solar and arc spectra.

12. From 102 lines Albrecht finds a small Larmor effect which disappears when 3 lines giving inconsistently large values are omitted.

13. The later recognized systematic errors in the published data on pressure-shift invalidate conclusions based upon them.

14. The correspondence between the errors in the Rowland wave-lengths of lines with close companions and the displacements noted by Albrecht is so complete that it appears a practical certainty that these displacements are another measurement of the errors.

15. They therefore furnish no valid evidence that the relative positions of the Fraunhofer lines are systematically displaced by mutual influence. On the other hand, the sun-arc displacements (370 lines) and the relative separations of the components of 45 close pairs in solar and arc spectra indicate that, within the limits of error, evidence of mutual influence is absent from the general solar spectrum, and in so far as mutual influence is a necessary corollary of anomalous dispersion evidence for the existence of the latter is also absent.

MOUNT WILSON SOLAR OBSERVATORY

August 1916

REVIEWS

A Meteorological Treatise on the Circulation and Radiation in the Atmospheres of the Earth and of the Sun. By F. H. BIGELOW. New York: John Wiley & Sons, Inc., 1915. Pp. xi+431. 78 figures. \$5.00.

At present there is a lack of textbooks on meteorology in which an adequate account of the mathematical theory is given, and a student who is laboriously endeavoring to piece together the fragments of theory which are scattered in the numerous periodicals will warmly welcome a mathematical treatise written by an experienced hand. Professor Bigelow's book contains not only an account of work which has become classical; it is devoted chiefly to a presentation of the results of his own researches and deals with a large number of problems which are of present-day interest. It deserves, then, to be reviewed as an important contribution to the elucidation of these questions.

The author first points out that Boyle's law for a perfect gas in which the pressure P , the density ρ , and the absolute temperature T are connected by the relation $P = \rho RT$ does not agree with observations made at different heights unless R is supposed to vary with the height above sea-level. The physical reason for this variation is indicated on p. 80, where it is stated that air is not an ideal gas but rather a mixture of gases which are undergoing rapid changes of condition through variations in the heat contents by insolation and radiation. The processes taking place in the atmosphere are thus not generally adiabatic, and in the formula $dT = -adz$, for the variation of the temperature with the height, the non-adiabatic gradient a is generally less than the adiabatic gradient a_0 .

The formulae of the kinetic theory of gases for a mixture of several gases are given in chap. i: they indicate that R varies with the composition of the air. This idea of the variation of the gas coefficient R is fundamental in Mr. Bigelow's thermodynamical theory. The relation $R = C_p - C_v$, between R and the specific heats at constant pressure and constant volume, is adopted as usual and the ratio k of the two specific heats is assumed to be constant, although strictly this also varies slightly

with the composition of the air, as is indicated by Capstick's formula (*Philosophical Transactions*, 1894), which has been applied to a mixture of air and water-vapor by G. Schweikert (*Annalen der Physik*, 48, 593, 1915) and by Leduc's modified formula (*Comptes Rendus*, 160, 316, 1915), which holds when the gases are not perfect.

The author proceeds on the foregoing assumptions to develop some useful formulae for the computation of P , ρ , and R from the observed values of T and gives tables to illustrate his methods of computation. In the derivation of the non-adiabatic formulae, in which $n = a_0/a$ occurs as a variable, there are some points which are a little obscure. The approximation

$$\int \frac{dP}{\rho} = \frac{P_1 - P_0}{\rho_{10}}, \quad \rho_{10} = \frac{1}{2}(\rho_1 + \rho_0)$$

which is made on p. 63 requires justification, and on p. 58 the term involving $\log T_0$ seems to be dropped in the transition from equation (188) to (189).

By introducing a term representing the heat lost by radiation into the hydrodynamical equation for the kinetic energy, and combining this equation with his thermodynamical equations, Mr. Bigelow obtains a number of interesting relations which he applies to the study of radiation and circulation in the atmosphere; the radiation being expressed by a formula analogous to that occurring in Stefan's law, except that the index is not equal to four.

With regard to the isothermal layer Mr. Bigelow says: "The principal fact to be explained is the slow rate of loss of heat in the convective region as compared with that in the isothermal region." To explain this the author makes use of his equation $Q'_1 - Q'_0 = C_{p10}(T_1 - T_0)$ for the evolution of heat during the vertical convection of air from a place where the temperature is T_0 to a place where it is T_1 ; C_{p10} here denotes the mean value of the specific heat at constant pressure. Thus if $Q_1 - Q_0$ is the natural loss of heat by radiation without convection, this quantity represents the loss of heat in the isothermal region, but in the convective region the loss of heat is $(Q_1 - Q_0) - (Q'_1 - Q'_0)$. The author illustrates his ideas by discussing the diurnal convection and the semi-diurnal waves in the lower strata and the thermodynamic structure of cyclones and anticyclones.

The hydrodynamical equations of motion are next discussed and the author finds that two interesting types of vortices, which he describes as funnel-shaped and dumb-bell-shaped, may be specified by means

of the current functions $\psi = A\hat{\omega}z$ and $\psi = A\hat{\omega}^2 \sin az$, z and $\hat{\omega}$ being cylindrical co-ordinates and A and a constants. The motions in certain waterspouts, tornadoes, and cyclones which are described in chap. iv appear to resemble closely these two types of vortex motion. In the next two chapters the author deals with a variety of topics, such as the measurement of the intensity of solar radiation by means of the pyrheliometer and bolometer, the ionization of the atmosphere and measurement of conductivity, the diurnal convection in the earth's atmosphere, the diurnal variations of the meteorological, electrical, and magnetic elements, the laws of evaporation, polarization of sunlight, and solar physics. The author describes and discusses the theory of some of the instruments used in these branches of physics. His remarks on Bouguer's formula of depletion are interesting, the author being inclined to the view that the true "solar constant" is more than four calories per square centimeter per minute instead of two, the number which is usually adopted. The treatment of atmospheric electricity is to some extent unorthodox.

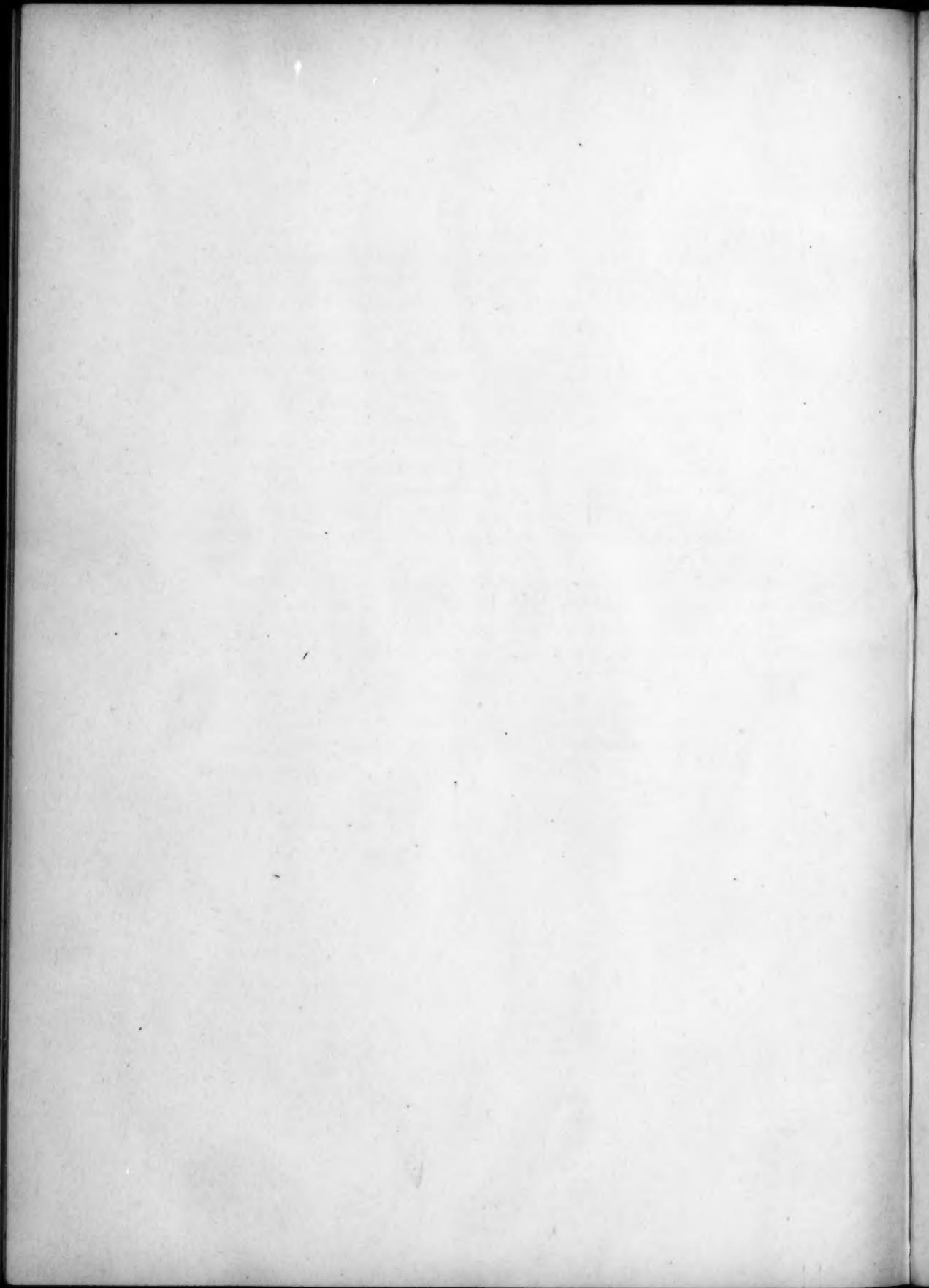
It is impossible to do justice in a short review to a work which covers such a vast field of research. The numerous tables and instructions for making computations will be invaluable to the student in observational work. There are also many interesting diagrams, all of which are well drawn.

In his tables of constants the author is generally up to date. In Table 7, p. 31, the formula for the conduction coefficient, viz., $L = 1.667 \eta C$, may perhaps need correction (cf. S. Chapman, *Philosophical Transactions [A]*, 211, 462, 1912).

H. BATEMAN

JOHNS HOPKINS UNIVERSITY
May 17, 1916

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